UCDAVIS

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Logarithms December 1, 2016

Logarithms

Outline

- Last Time: Exponents
- Review Quiz
- What is Logarithm: Exponent and Logarithm
- What is Logarithm: Plot of Exponent and Logarithm
- Main Formulas
- Applications
- History
- QUIZ

Last Time: Exponents

- Integer exponent: b^n

$3^1 = 3$, $3^2 = 9$, $3^3 = 27$

$3^{-1} = \frac{1}{3}$, $3^{-2} = \frac{1}{9}$, $3^{-3} = \frac{1}{27}$

$3^0 = 1$

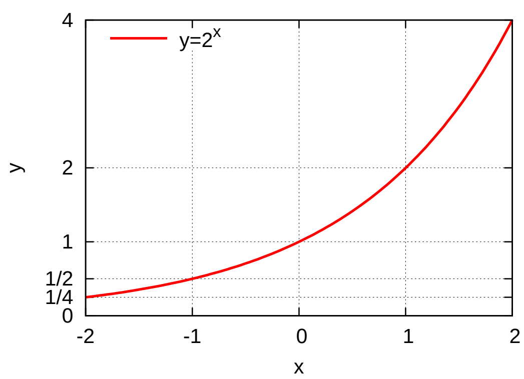
- Fractional exponent: $b^{\frac{m}{n}} = \sqrt[n]{b^m}$

- Arbitrary real exponent: b^r

- Decimal exponent: 10^x

- Natural exponent: e^x, $e = 2.718281828...$

- Graph of exponential function



Review QUIZ

Calculate:

$$(1) \quad 2^4 =$$

$$(2) \quad 2^6 =$$

$$(3) \quad 4^3 =$$

$$(4) \quad 2^{\frac{1}{2}} =$$

$$(5) \quad 2^{\frac{3}{2}} =$$

$$(6) \quad 10^2 =$$

$$(7) \quad 10^3 =$$

Solve equation:

$$(8) \quad 10^{\mathrm{x}} = 100$$

$$(9) \quad 10^{\mathrm{x}} = 1000$$

$$(10) \quad 2^{\mathrm{x}} = 16$$

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying b by itself n times

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$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = ?$ - multiplying 2 by itself 4 times

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \; = \; a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \; = \; 16$ - multiplying 2 by itself 4 times

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \;=\; a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \;=\; 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \;=\; a$

b known, n known, a unknown. Problem: find a

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \; = \; a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \; = \; 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \; = \; a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times
$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times
Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$
b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:
b known, n unknown, a known. Problem: find n
How many times to multiply base b by itself to get a?
$2^n = 2 \cdot 2... \cdot 2 \ = \ 16$ - how many times?

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

$2^n = 2 \cdot 2 ... \cdot 2 \ = \ 16 \quad$ 4 times

What is logarithm

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Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

$2^n = 2 \cdot 2 ... \cdot 2 \ = \ 16 \quad 4$ times

n = 4

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

$2^n = 2 \cdot 2 ... \cdot 2 \ = \ 16 \quad 4$ times

n = 4

$\mathbf{log_2} 16 \ = 4$

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

$2^n = 2 \cdot 2... \cdot 2 \ = \ 16 \quad 4$ times

$\mathrm{n} \ = \ 4$

$\mathbf{log_2} 16 \ = 4$

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \; = \; a$ - multiplying base b by itself n times

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Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \; = \; a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

$2^n = 2 \cdot 2... \cdot 2 \; = \; 16 \quad 4$ times

n = 4

$\mathbf{log}_2 16 \; = 4$

$b^n = b \cdot b \cdot ... \cdot b \; = \; a$ - multiplying base b by itself n times

$\mathbf{log}_b a \; = \mathbf{?}$

What is logarithm

- Exponentiation:

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 \ = \ 16$ - multiplying 2 by itself 4 times

Exponentiation: $b^n = b \cdot b \cdot ... \cdot b \ = \ a$

b known, n known, a unknown. Problem: find a

Logarithm: Inverse Problem:

b known, n unknown, a known. Problem: find n

How many times to multiply base b by itself to get a?

$2^n = 2 \cdot 2... \cdot 2 \ = \ 16 \quad 4$ times

n = 4

$\mathbf{log}_2 16 \ = 4$

$b^n = b \cdot b \cdot ... \cdot b \ = \ a$ - multiplying base b by itself n times

$\mathbf{log}_b a \ = \mathrm{n}$

What is logarithm

Summary

$\mathbf{b}^n = \mathbf{a}$

Means the same as

$\mathbf{log}_b \ \mathbf{a} = \mathbf{?}$

What is logarithm

Summary

$$\mathbf{b}^n = \mathbf{a}$$

Means the same as

$$\mathbf{log}_b\ \mathbf{a} = \mathbf{n}$$

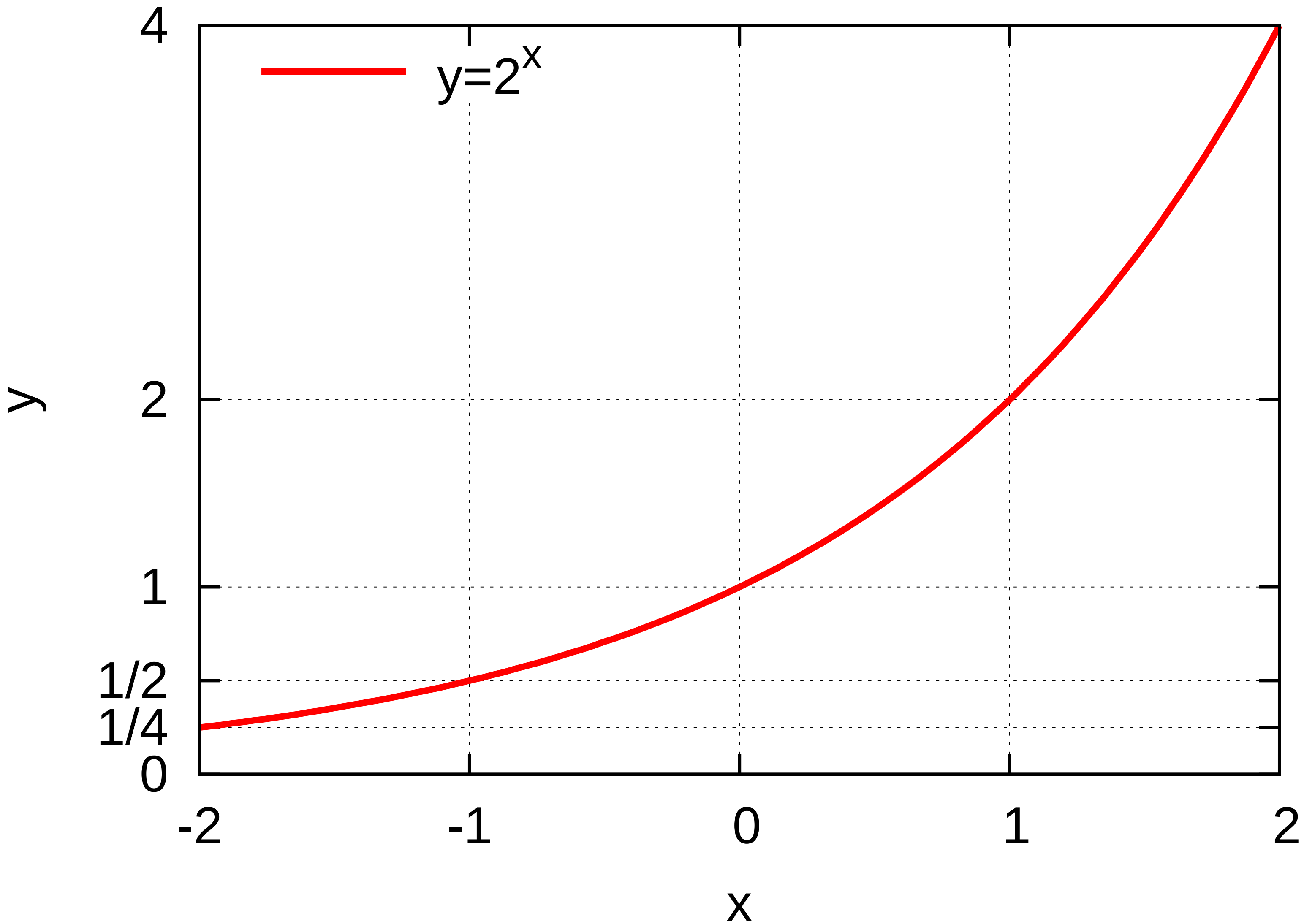
What is logarithm: Plot

Cannot solve everything in integers.
Need to find logarithm for any positive real number.

What is logarithm: Plot

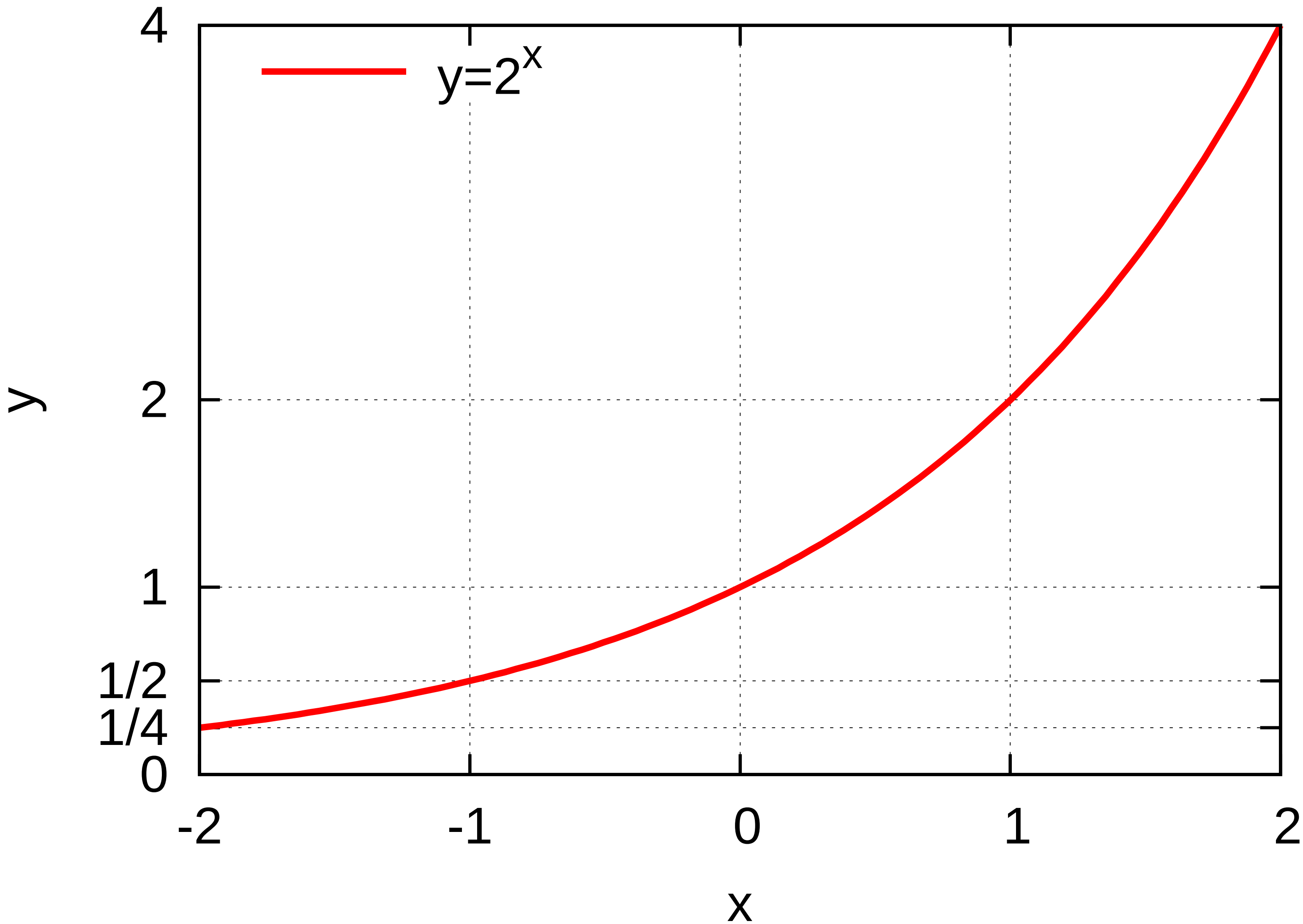
Exponent $\mathbf{y} = \mathbf{b}^x$

Plot of Exponent $\mathbf{y} = \mathbf{2}^x$



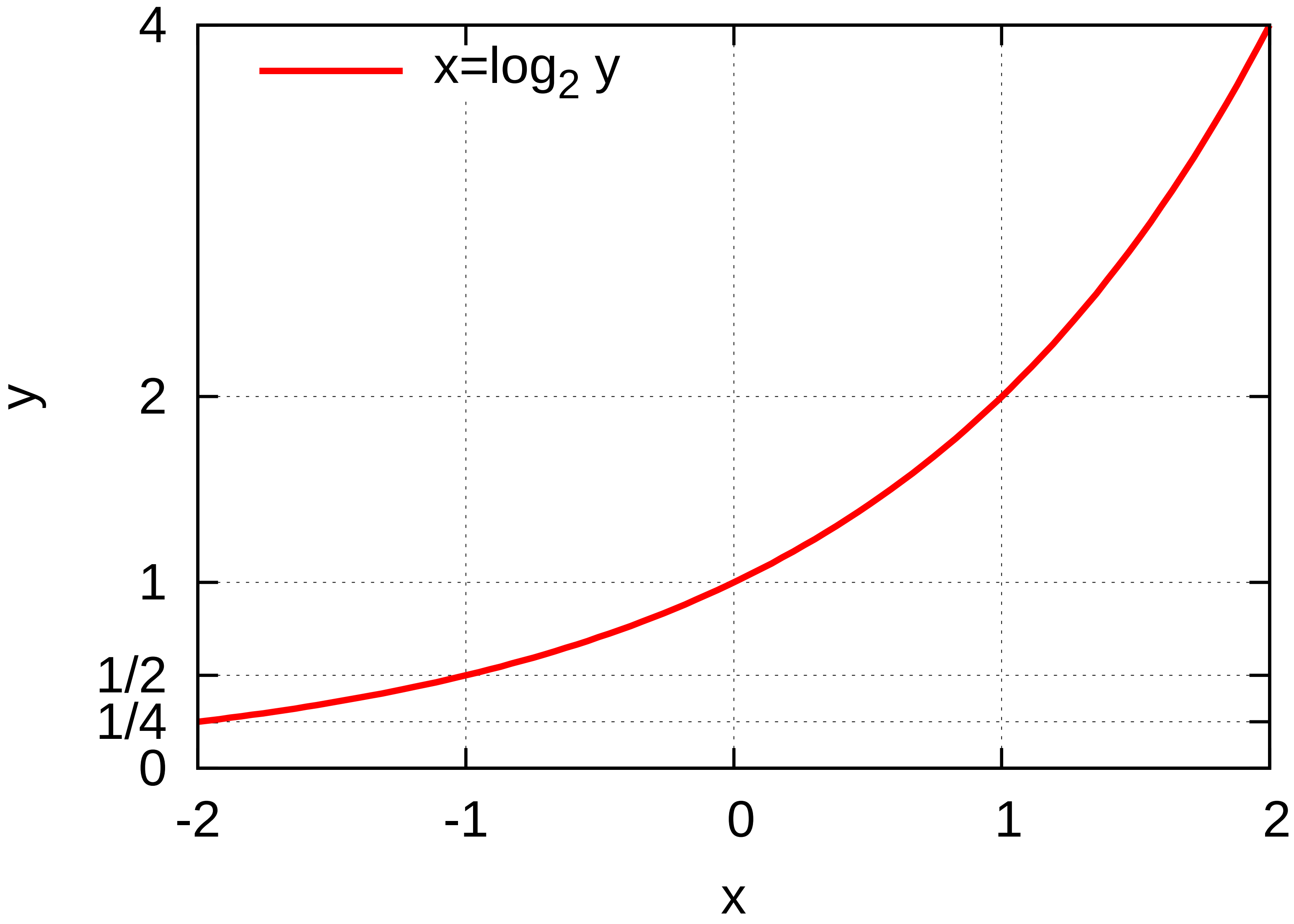
What is logarithm: Plot

$\mathbf{y = 2^x}$ means the same as $\mathbf{x = ?}$



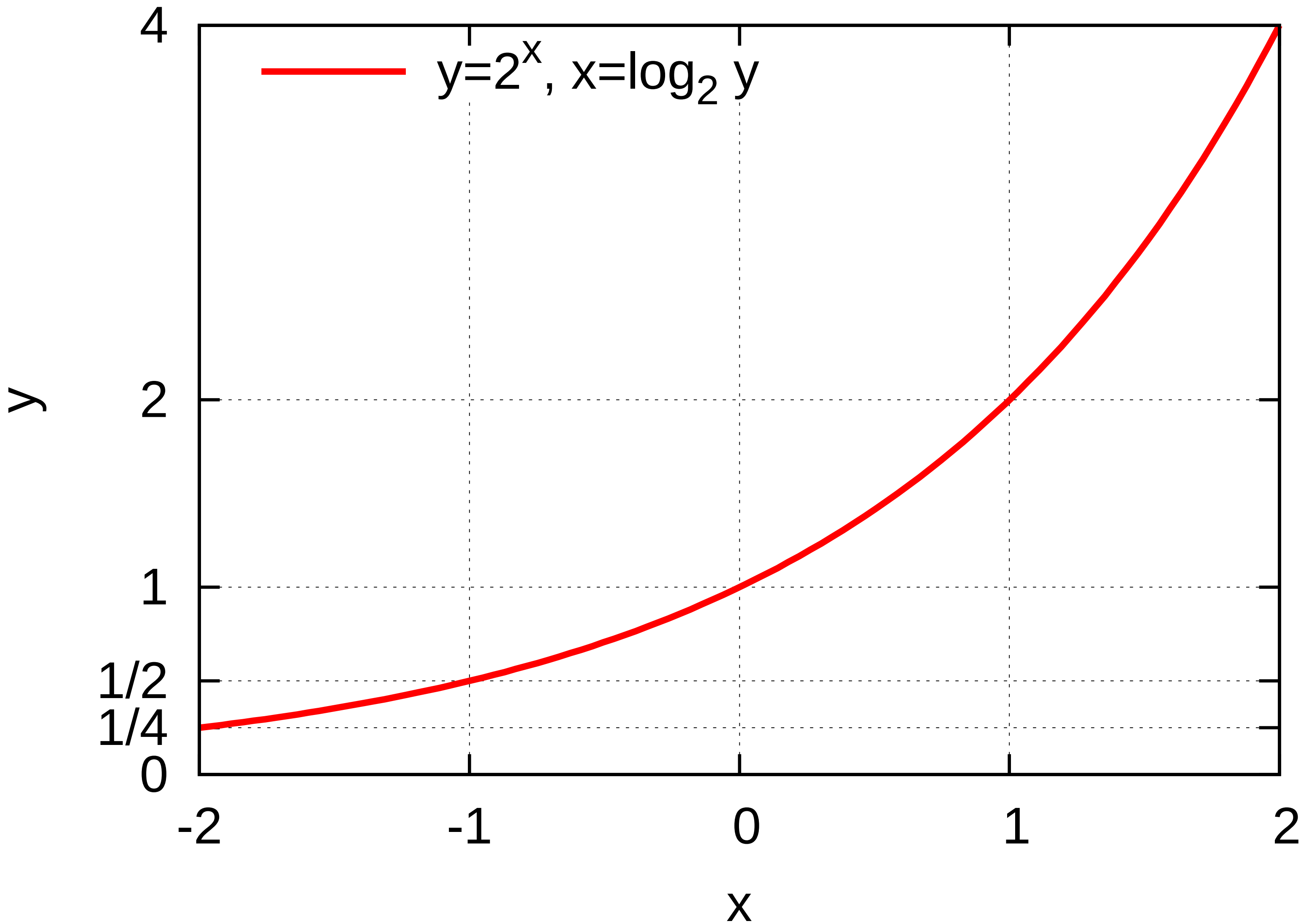
What is logarithm: Plot

$\mathbf{y = 2^{x}}$ means the same as $\mathbf{x = \log_2 y}$



What is logarithm: Plot

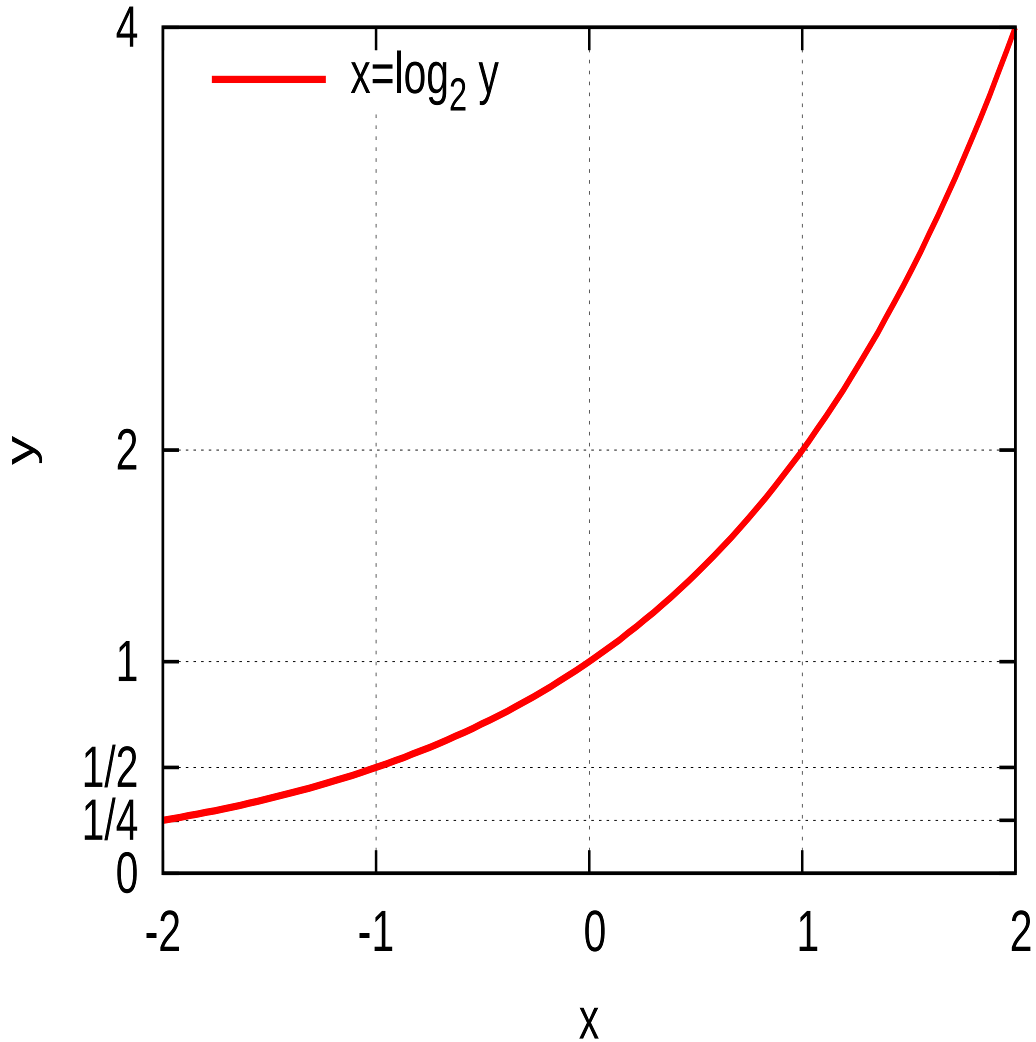
Plot of $\mathbf{y} = \mathbf{2}^x$ is also plot of $\mathbf{x} = \mathbf{log}_2\ \mathbf{y}$



What is logarithm: Plot

$\mathbf{x = \log_2 y}$

Usually argument of a function is **x**, and is horizontal.
Let's rename **x** and **y**, and flip axes.

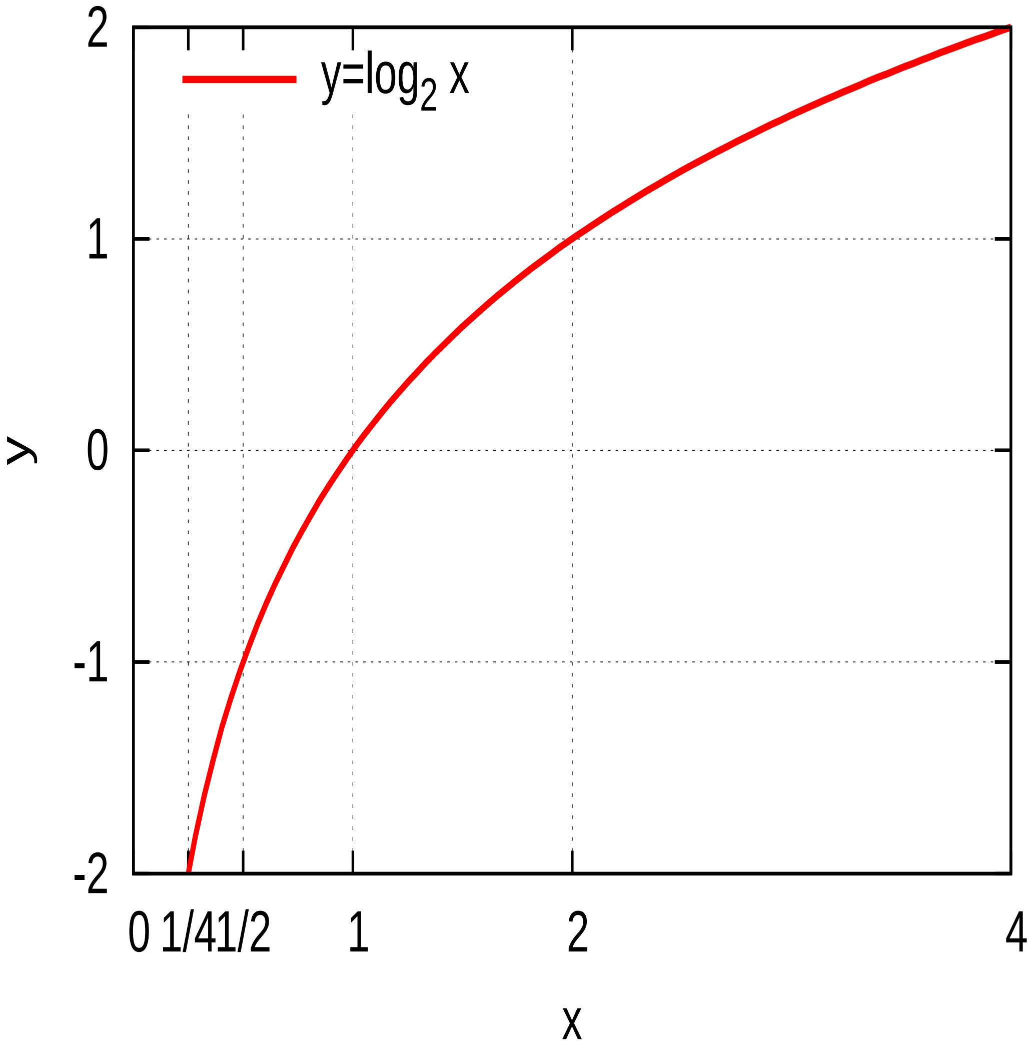


What is logarithm: Plot

$\mathbf{y = \log_2 x}$

Usually argument of a function is **x**, and is horizontal.

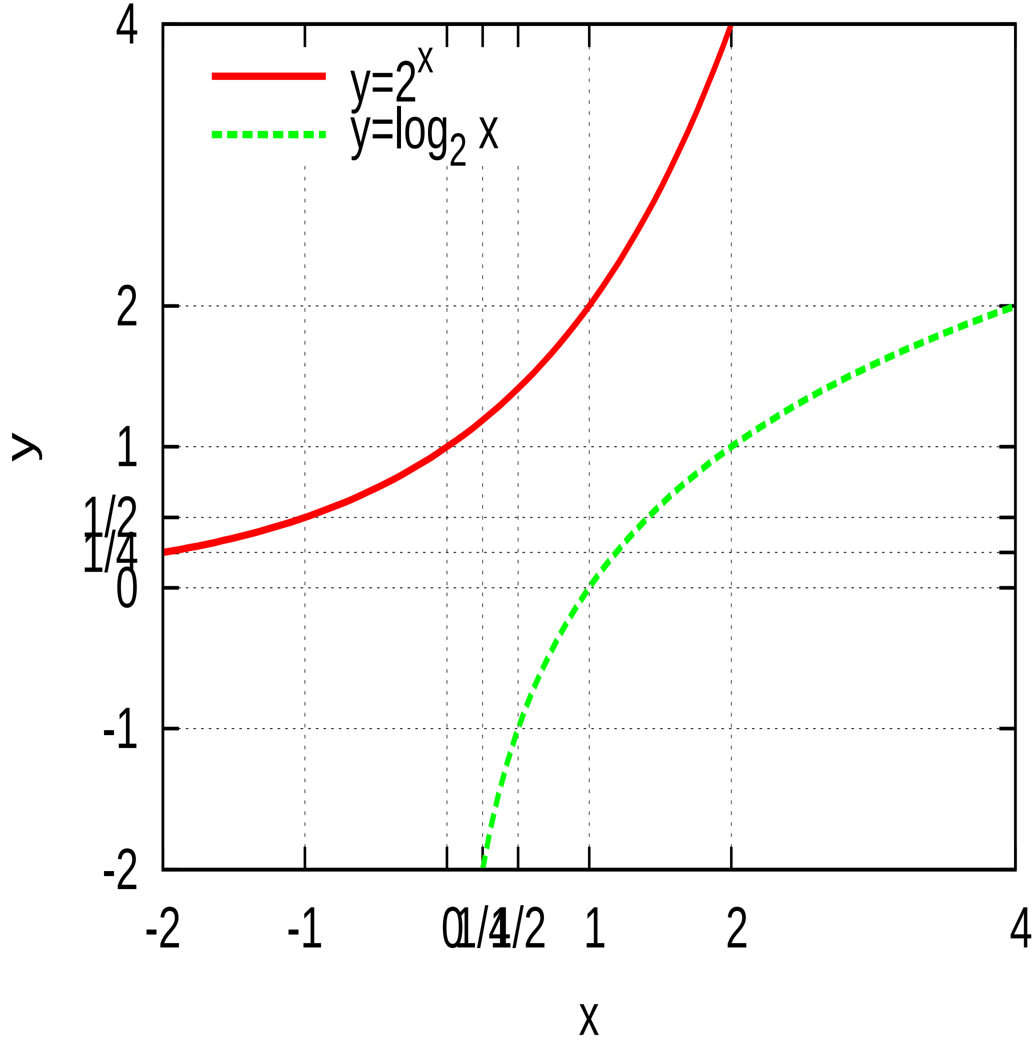
Let's rename **x** and **y**, and flip axes. Now plot is:



What is logarithm: Plot

$\mathbf{y = 2^x}$ and $\mathbf{y = \log_2 x}$ together

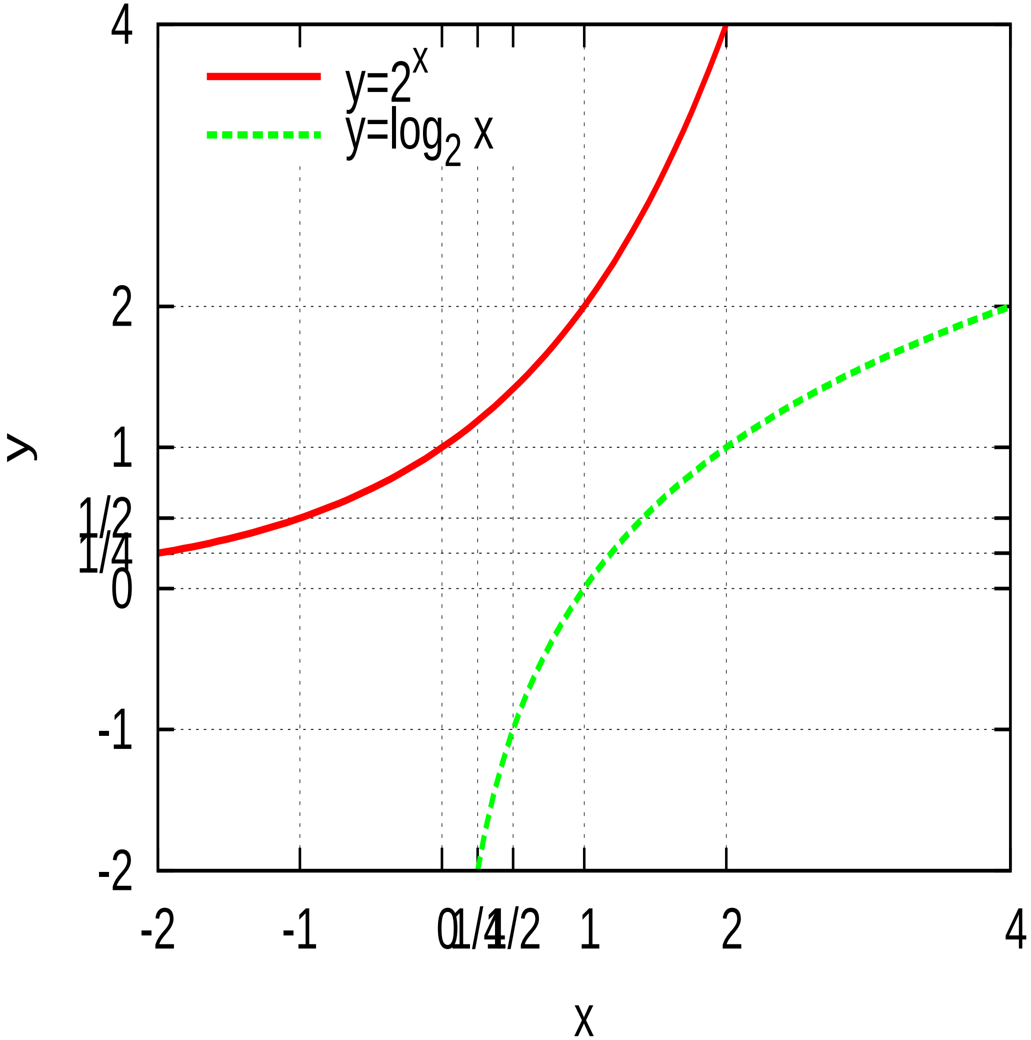
What we may say about relation between plots?



What is logarithm: Plot

$\mathbf{y = 2^x}$ and $\mathbf{y = \log_2 x}$ together

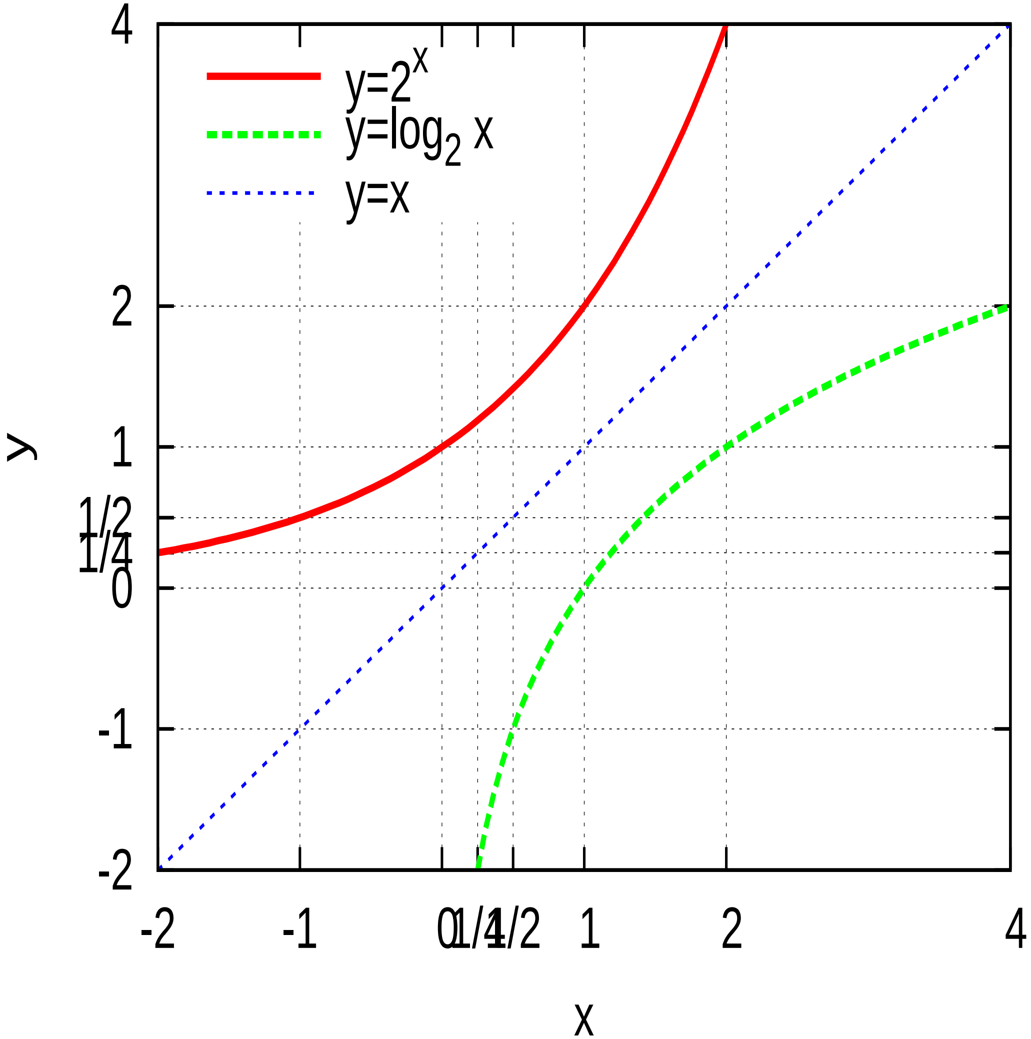
The plots are the same plot flipped over the line (what line?)



What is logarithm: Plot

$\mathbf{y = 2^{x}}$ and $\mathbf{y = \log_2 x}$ together

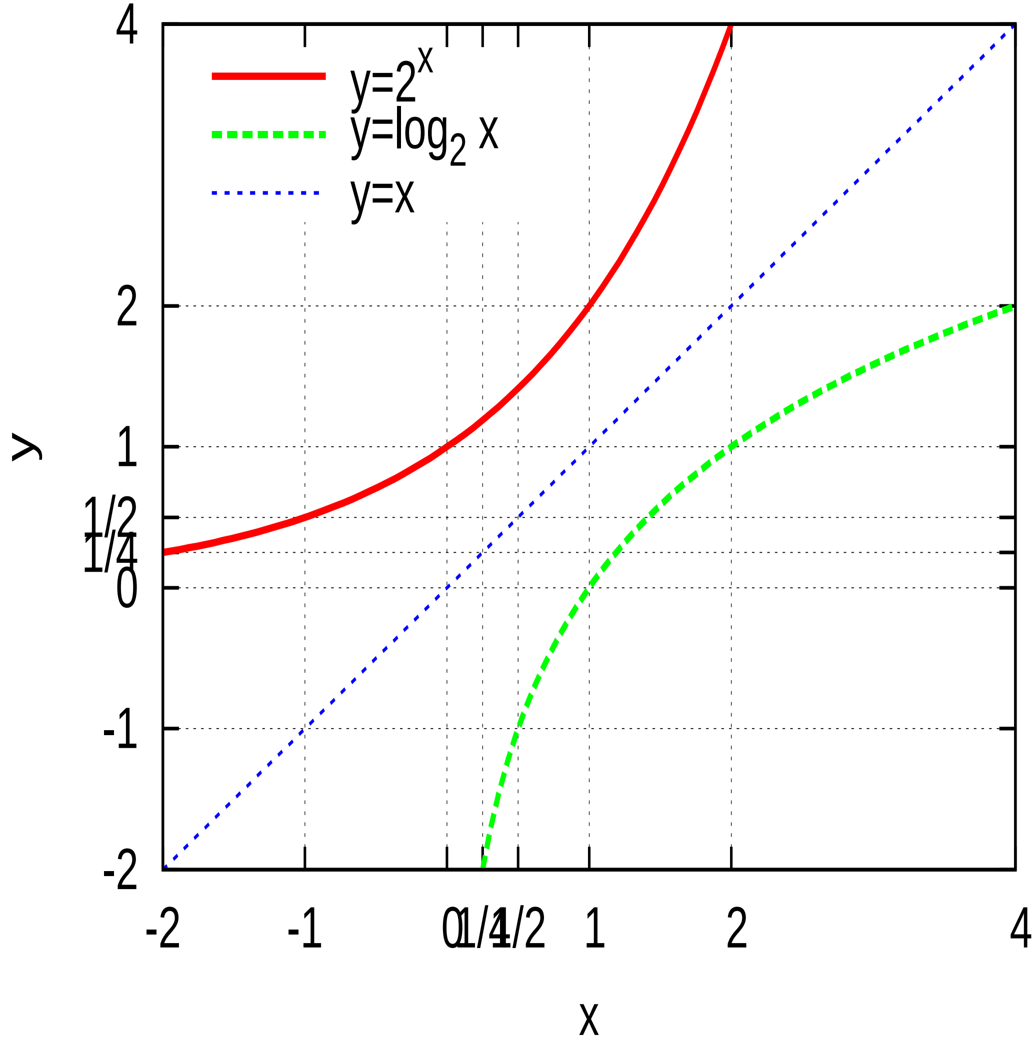
The plots are the same plot flipped over the line $\mathbf{y = x}$



What is logarithm: Plot

$\mathbf{y = 2^x}$ and $\mathbf{y = \log_2 x}$ together

The plots are symmetric relatively to the line $\mathbf{y = x}$



Main Formulas

As we just learned,

$\mathbf{y} = \mathbf{b}^x$

means the same as

$\mathbf{x} = \mathbf{?}$

Main Formulas

As we just learned,

$$\mathbf{y} = \mathbf{b}^{x}$$

means the same as

$$\mathbf{x} = \mathbf{log}_{b}\ \mathbf{y}$$

Main Formulas

As we just learned,

$$\mathbf{y} = \mathbf{b}^x$$

means the same as

$$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$$

Therefore, every property of exponent

$$\mathbf{y} = \mathbf{b}^x$$

corresponds to some property of logarithm

$$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$$

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{?}$

Main Formulas

Every property of exponent

$\mathbf{y = b}^x$

corresponds to some property of logarithm

$\mathbf{x = \log}_b\ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Let's name

$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.

Then

$\mathbf{b}^{x+y} = \mathbf{?}$.

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Let's name

$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.

Then

$\mathbf{b}^{x+y} = \mathbf{c}\ \mathbf{d}$.

Write last three formulas in terms of logarithms:

$\mathbf{x} = \mathbf{?}$

$\mathbf{y} = \mathbf{?}$

$\mathbf{x+y} = \mathbf{?}$

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Let's name

$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.

Then

$\mathbf{b}^{x+y} = \mathbf{c}\ \mathbf{d}$.

Write last three formulas in terms of logarithms:

$\mathbf{x} = \mathbf{log}_b \mathbf{c}$

$\mathbf{y} = \mathbf{?}$

$\mathbf{x+y} = \mathbf{?}$

Main Formulas

Every property of exponent
$\mathbf{y} = \mathbf{b}^x$
corresponds to some property of logarithm
$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$

Recall property of exponent:
$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$
Let's write the formula in terms of logarithms.
Let's name
$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.
Then
$\mathbf{b}^{x+y} = \mathbf{c}\ \mathbf{d}$.
Write last three formulas in terms of logarithms:
$\mathbf{x} = \mathbf{log}_b\ \mathbf{c}$
$\mathbf{y} = \mathbf{log}_b\ \mathbf{d}$
$\mathbf{x+y} = \mathbf{?}$

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b \ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Let's name

$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.

Then

$\mathbf{b}^{x+y} = \mathbf{c} \ \mathbf{d}$.

Write last three formulas in terms of logarithms:

$\mathbf{x} = \mathbf{log}_b \ \mathbf{c}$

$\mathbf{y} = \mathbf{log}_b \ \mathbf{d}$

$\mathbf{x{+}y} = \mathbf{log}_b \ (\mathbf{cd})$

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Let's name

$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.

Then

$\mathbf{b}^{x+y} = \mathbf{c}\ \mathbf{d}$.

Write last three formulas in terms of logarithms:

$\mathbf{x} = \mathbf{log}_b\ \mathbf{c}$

$\mathbf{y} = \mathbf{log}_b\ \mathbf{d}$

$\mathbf{x+y} = \mathbf{log}_b\ \mathbf{(cd)}$

Let's rewrite in the last formula **x** and **y** in terms of logarithms:

Main Formulas

Every property of exponent

$\mathbf{y} = \mathbf{b}^x$

corresponds to some property of logarithm

$\mathbf{x} = \mathbf{log}_b\ \mathbf{y}$

Recall property of exponent:

$\mathbf{b}^x \cdot \mathbf{b}^y = \mathbf{b}^{x+y}$

Let's write the formula in terms of logarithms.

Let's name

$\mathbf{b}^x = \mathbf{c}$, $\mathbf{b}^y = \mathbf{d}$.

Then

$\mathbf{b}^{x+y} = \mathbf{c}\ \mathbf{d}$.

Write last three formulas in terms of logarithms:

$\mathbf{x} = \mathbf{log}_b\ \mathbf{c}$

$\mathbf{y} = \mathbf{log}_b\ \mathbf{d}$

$\mathbf{x+y} = \mathbf{log}_b\ (\mathbf{cd})$

Let's rewrite in the last formula **x** and **y** in terms of logarithms:

$\mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d} = \mathbf{log}_b\ (\mathbf{cd})$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

More than 2 multipliers

$$\mathbf{log}_b\ (\mathbf{cdef}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

More than 2 multipliers

$$\mathbf{log}_b\ (\mathbf{cdef}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d} + \mathbf{log}_b\ \mathbf{e} + \mathbf{log}_b\ \mathbf{f}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

More than 2 multipliers

$$\mathbf{log}_b\ (\mathbf{cdef}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d} + \mathbf{log}_b\ \mathbf{e} + \mathbf{log}_b\ \mathbf{f}$$

The same multipliers

$$\mathbf{log}_b\ (\mathbf{a\ a\ a\ a}) = \mathbf{log}_b\ (\mathbf{a}^4) = ?$$

Main Formulas

$$\log_b (cd) = \log_b c + \log_b d$$

More than 2 multipliers

$$\log_b (cdef) = \log_b c + \log_b d + \log_b e + \log_b f$$

The same multipliers

$$\log_b (a\ a\ a\ a) = \log_b (a^4) = 4 \log_b a$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

More than 2 multipliers

$$\mathbf{log}_b\ (\mathbf{cdef}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d} + \mathbf{log}_b\ \mathbf{e} + \mathbf{log}_b\ \mathbf{f}$$

The same multipliers

$$\mathbf{log}_b\ (\mathbf{a\ a\ a\ a}) = \mathbf{log}_b\ (\mathbf{a}^4) = \mathbf{4\ log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

More than 2 multipliers

$$\mathbf{log}_b\ (\mathbf{cdef}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d} + \mathbf{log}_b\ \mathbf{e} + \mathbf{log}_b\ \mathbf{f}$$

The same multipliers

$$\mathbf{log}_b\ (\mathbf{a\ a\ a\ a}) = \mathbf{log}_b\ (\mathbf{a}^4) = \mathbf{4\ log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n\ log}_b\ \mathbf{a}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

Main Formulas

Example

$$\mathbf{\log_{10}(10000) = ?}$$

Main Formulas

Example

$$\log_{10}(10000) = \log_{10}(10^4) = 4\log_{10}(10) = 4$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{1}{a}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{1}{a}) = \mathbf{log}_b\ (a^{-1}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{1}{a}) = \mathbf{log}_b\ (a^{-1}) = \text{-}\mathbf{log}_b\ \mathbf{a}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n\ log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{1}{a}) = \mathbf{log}_b\ (a^{-1}) = \textbf{-log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{c}{d}) = \mathbf{log}_b\ (c \cdot d^{-1}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{1}{a}) = \mathbf{log}_b\ (a^{-1}) = \mathbf{-log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\tfrac{c}{d}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ (d^{-1}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\frac{1}{a}) = \mathbf{log}_b\ (a^{-1}) = \text{-}\mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\frac{c}{d}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ (d^{-1}) = \mathbf{log}_b\ \mathbf{c}\ \text{-}\ \mathbf{log}_b\ \mathbf{d}$$

Main Formulas

$$\mathbf{log}_b\ \left(\frac{c}{d}\right) = \mathbf{log}_b\ \mathbf{c}\ \textbf{-}\ \mathbf{log}_b\ \mathbf{d}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\sqrt[n]{a}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\sqrt[n]{a}) = \mathbf{log}_b\ (a^{\frac{1}{n}}) = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\sqrt[n]{a}) = \mathbf{log}_b\ (a^{\frac{1}{n}}) = \tfrac{1}{n}\ \mathbf{log}_b\ \mathbf{a}$$

Main Formulas

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Main Formulas

Recall property of exponent:

$(\mathbf{c}^{\mathbf{x}})^{\mathbf{y}} = ?$

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$\log_b a = \frac{\log_c a}{\log_c b}$

Main Formulas

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Example

$$\mathbf{log}_{(b^n)}\ \mathbf{a} = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_b\ \mathbf{a} = \frac{\log_{\mathbf{c}} \mathbf{a}}{\log_{\mathbf{c}} \mathbf{b}}$$

Example

$$\mathbf{log}_{(b^n)}\ \mathbf{a} = \frac{\log_{\mathbf{b}} \mathbf{a}}{\log_{\mathbf{b}} (\mathbf{b^n})} =$$

Main Formulas

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$$\mathbf{log}_{(b^n)}\ \mathbf{a} = \frac{\log_{\mathbf{b}} \mathbf{a}}{\log_{\mathbf{b}} (\mathbf{b}^{\mathbf{n}})} = \frac{\log_{\mathbf{b}} \mathbf{a}}{\mathbf{n}}$$

Main Formulas

$$\mathbf{log}_b\ \mathbf{a} = \frac{\log_{\mathbf{c}} \mathbf{a}}{\log_{\mathbf{c}} \mathbf{b}}$$

$$\mathbf{log}_{(b^n)}\ \mathbf{a} = \frac{\mathbf{1}}{\mathbf{n}} \log_{\mathbf{b}} \mathbf{a}$$

Main Formulas

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Compare

$$\log_b(a^n) = n \log_b a$$

Main Formulas

$$\mathbf{log}_{(b^n)}\ \mathbf{a} = \frac{\mathbf{1}}{\mathbf{n}} \log_{\mathbf{b}} \mathbf{a}$$

Example

$$\mathbf{log}_{1000}\ \mathbf{10} = \mathbf{?}$$

Main Formulas

$$\mathbf{log}_{(b^n)}\ \mathbf{a} = \frac{\mathbf{1}}{\mathbf{n}} \log_{\mathbf{b}} \mathbf{a}$$

Example

$$\mathbf{log}_{1000}\ \mathbf{10} = \mathbf{log}_{10^3}\ \mathbf{10} = \frac{\mathbf{1}}{\mathbf{3}} \log_{\mathbf{10}} \mathbf{10} = \frac{\mathbf{1}}{\mathbf{3}}$$

Main Formulas

$$\mathbf{log}_b\ (\mathbf{cd}) = \mathbf{log}_b\ \mathbf{c} + \mathbf{log}_b\ \mathbf{d}$$

$$\mathbf{log}_b\ (\tfrac{c}{d}) = \mathbf{log}_b\ \mathbf{c}\ \text{-}\ \mathbf{log}_b\ \mathbf{d}$$

$$\mathbf{log}_b\ (\mathbf{a}^n) = \mathbf{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ (\sqrt[n]{a}) = \frac{1}{n}\ \mathbf{log}_b\ \mathbf{a}$$

$$\mathbf{log}_b\ \mathbf{a} = \frac{\log_{\mathbf{c}} \mathbf{a}}{\log_{\mathbf{c}} \mathbf{b}}$$

Applications

Logarithmic scale

Earthquake Richter scale

Strength of earthquake $= \log_{10^{\frac{3}{2}}}(\text{released energy}) + (\text{some constant})$

Applications

Logarithmic scale

Sound intensity

Decibel sound intensity level $= \log_{10}(\text{sound intensity})/(10^{-12} W/m^2)$

Applications

Logarithmic scale

History of the Universe

Seconds after Big Bang	Period
10^{-45} to 10^{-40}	Planck Epoch
10^{-40} to 10^{-35}	
10^{-35} to 10^{-30}	Epoch of Grand Unification
10^{-30} to 10^{-25}	
10^{-25} to 10^{-20}	
10^{-20} to 10^{-15}	
10^{-15} to 10^{-10}	Electroweak Epoch
10^{-10} to 10^{-5}	
10^{-5} to 10^{0}	Hadron Epoch
10^{0} to 10^{5}	Lepton Epoch
10^{5} to 10^{10}	Epoch of Nucleosynthesis
10^{10} to 10^{15}	Epoch of Galaxies
10^{15} to 10^{20}	

The present time is approximately 4.3×10^{17} seconds after the Big Bang; the Sun and Earth formed about 2×10^{17} seconds after the Big Bang. 10^{20} seconds is 3 trillion years (3×10^{12} years) in the future.

Applications

Mathematics: Number Theory

At numbers of magnitude **n**, the density of primes $\quad 1/\ln n$.

Applications

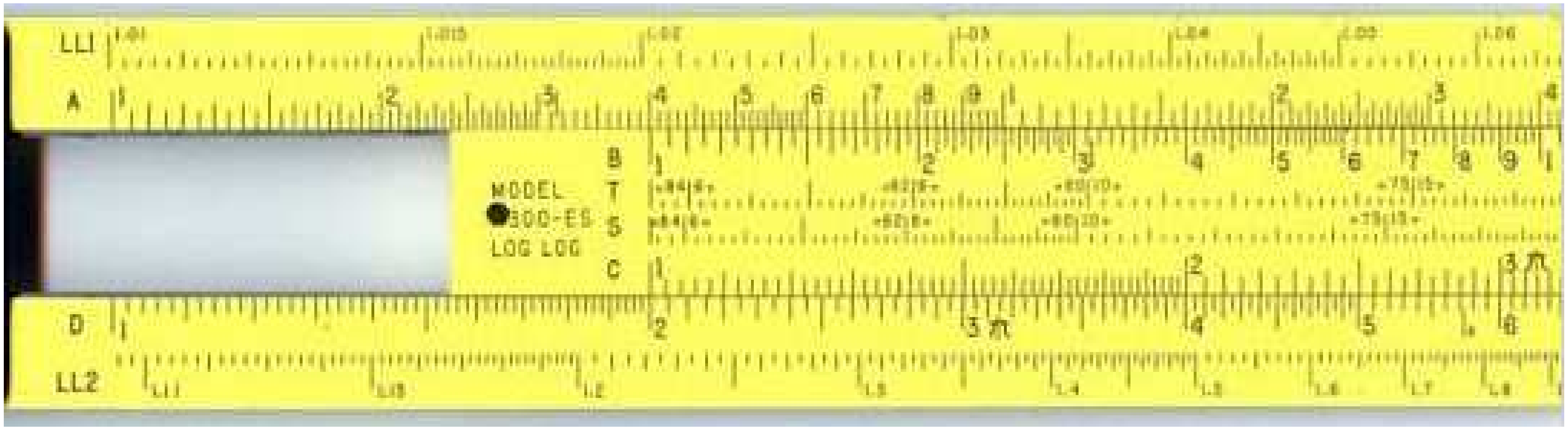
Physics: Entropy

$$S = -k_B \sum_i p_i \ln(p_i)$$

p_i - probability of state i, k_B - Boltzmann constant.

Applications

Slide rule

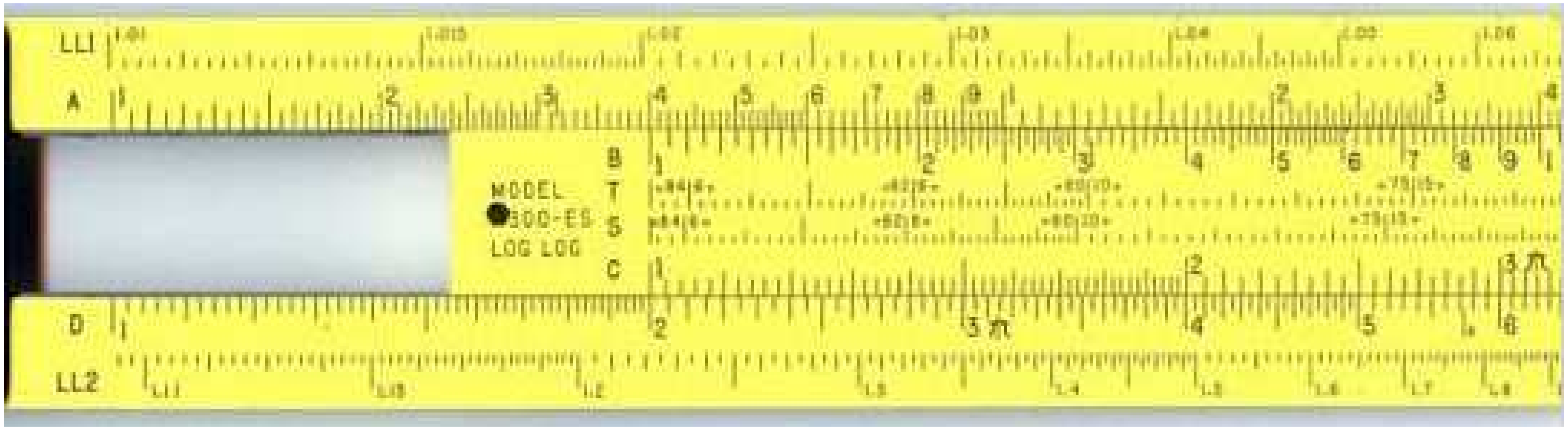


Most languages: **Calculating** rule

Russian and some other languages: **Logarithmic** rule

Applications

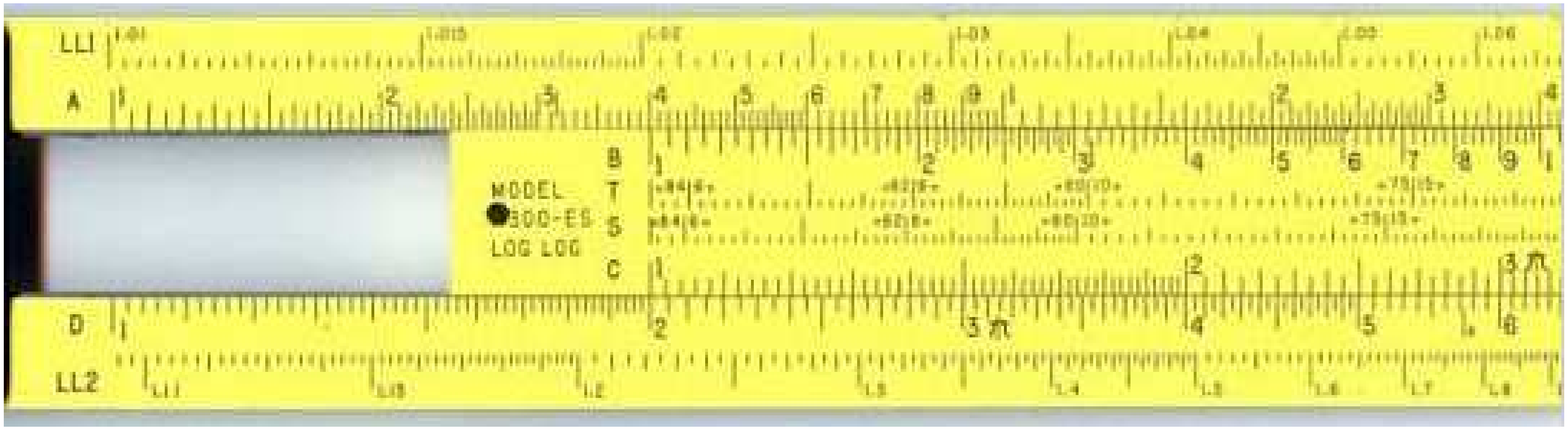
Slide rule



How to multiply on slide rule:
We need to multiply **a x b**, for example **2 x 3.**
Slide middle part to place **1** on part C against **a (= 2)** on part D.
Find **b (= 3)** on part C and read the respective number on part D. (= 6).
We performed multiplication
2 x 3 = 6

Applications

Slide rule



How does it work?

As a hint - recall that in Russian it is

Logarithmic rule

History

From Wikipedia:

John Napier of Merchiston (1550 - 1617)

The Napierian logarithms were published first in 1614.

Henry Briggs introduced common (base 10) logarithms, which were easier to use.

Tables of logarithms were published in many forms over four centuries.

The idea of logarithms was also used to construct the slide rule, which became ubiquitous in science and engineering until the 1970s.

QUIZ

Calculate:

$$
\begin{aligned}
&(1) \quad \log_{10} 1 = \\
&(2) \quad \log_{10} 1000 = \\
&(3) \quad \log_{100} 1000 = \\
&(4) \quad \log_{10} 0.001 = \\
&(5) \quad \log_{10} 2 + \log_{10} 5 = \\
&(6) \quad \log_{10} 8 + \log_{10} 125 = \\
&(7) \quad \log_{10} 80 - \log_{10} 8 =
\end{aligned}
$$

Solve equation:

$$
\begin{aligned}
&(8) \quad \log_{2} \mathrm{x} = 4 \\
&(9) \quad \log_{10} \mathrm{x} = -3 \\
&(10) \quad \log_{10} \mathrm{x} = -1
\end{aligned}
$$