UCDAVIS

Aleksander Zujev
Journal Club talk 2-02-2007

Introduction to Spin Hall Effect

J. E. Hirsch
Department of Physics, University of California, San Diego
PRL 83, 1834 (1999)

Outline

- Hall effect

- Anomalous Hall effect

- Spin Hall effect from Anomalous Hall effect

- Detection of Spin Hall effect

- Quantum Hall effect

Hall effect

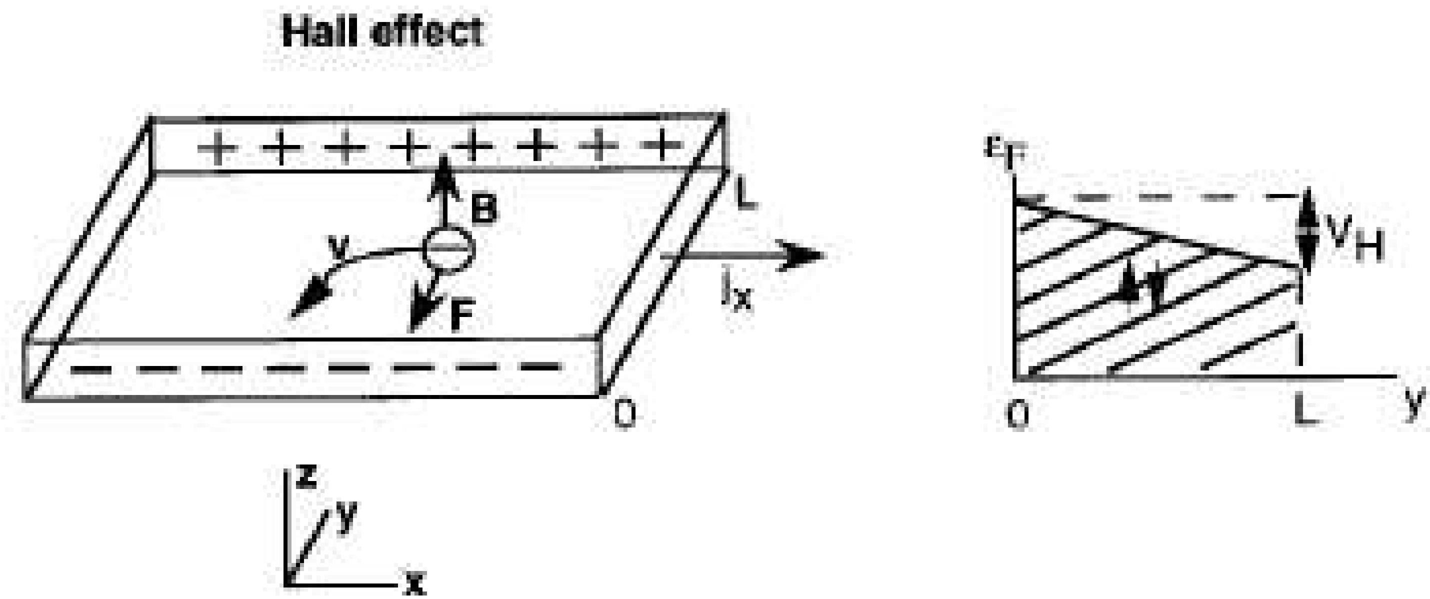
Electric current in magnetic field $\vec{B} \perp \vec{j}$

$\Rightarrow \vec{E}_H \perp \vec{j}, \vec{B}$

$E_H = \rho_H j$

$\rho_H = R_0 B$

R_0 - Hall coefficient. Easy to show: $R_0 = \frac{1}{en}$



Math

Electron moving in $\vec{B} \perp \vec{v}$.
Equilibrium between the forces:

$$evB = eE$$

$$j = nev$$

$$\Rightarrow E = j\frac{1}{ne}B$$

$$\rho_H = \frac{E}{j} = \frac{1}{ne}B$$

Resistivity tensor (2D, $\vec{B} \parallel \hat{z}$):

$$\rho = \begin{bmatrix} \rho_0 & \rho_H \\ -\rho_H & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & B/ne \\ -B/ne & \rho_0 \end{bmatrix}$$

Anomalous Hall effect

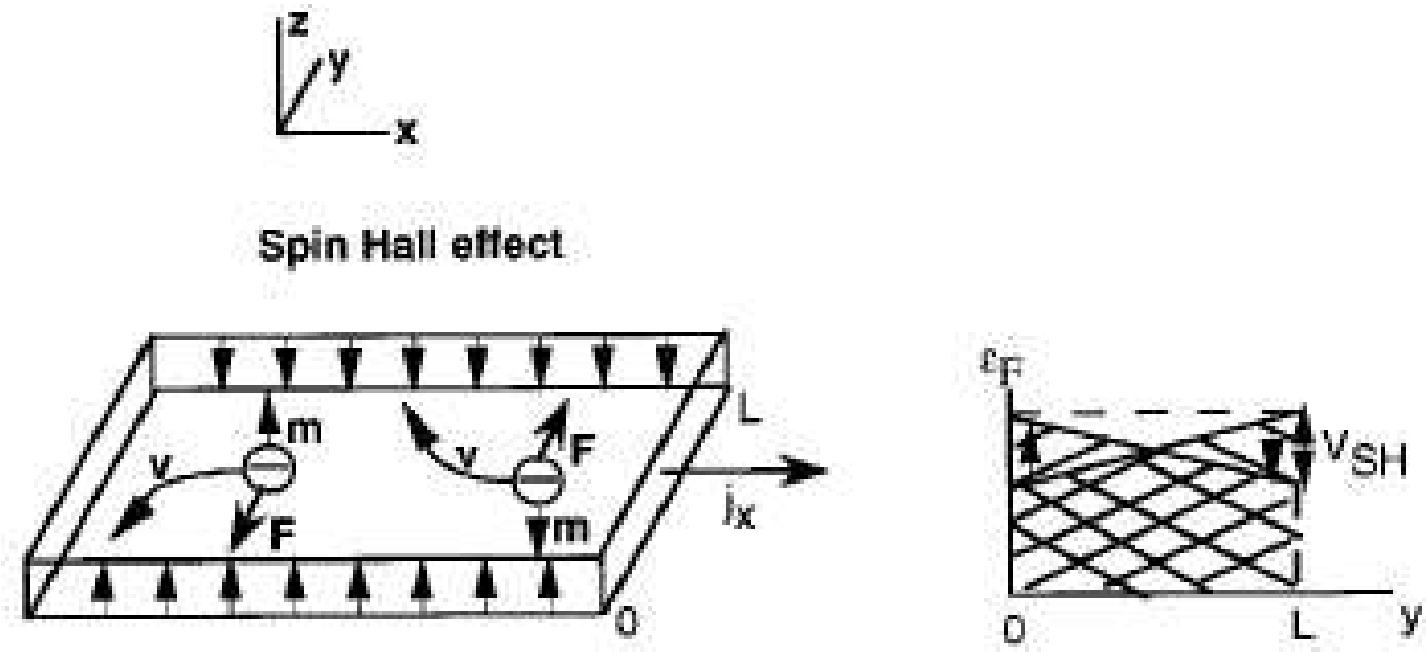
Electric current in magnetic field $\vec{B} \perp \vec{j}$
$\Rightarrow \vec{E}_H \perp \vec{j}, \vec{B}$
$E_H = \rho_H j$
$\rho_H = R_0 B + 4\pi R_s M$
M - magnetization
R_s - anomalous Hall coefficient

Origin: A few mechanisms proposed: skew scattering by impurities and phonons, "side jump" mechanism, other.

Exists in FM $\Rightarrow e^-$'s carrying spin and associated magnetic moment experience transverse force to their movement.

Spin Hall effect

No magnetization - e.g. paramagnet.
But scattering mechanisms causing Anomalous Hall effect exist:
$e^-_\uparrow$ scatter preferentially in one direction, $e^-_\downarrow$ in another.



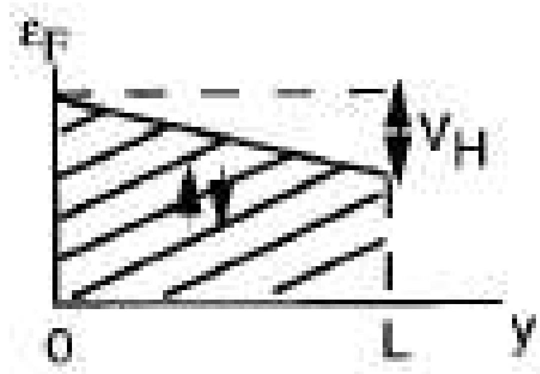
How to detect?

Hall effect:

Connect $y = 0$ and $y = L$:

e^- go $0 \;\to\; L$:

Electric current.



Spin Hall effect:

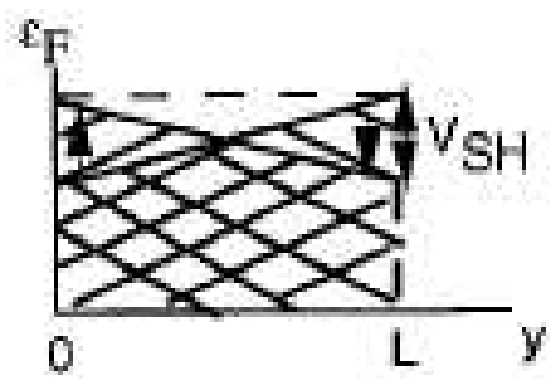
Connect $y = 0$ and $y = L$:

$e^-_\uparrow$ go $0 \;\to\; L$

$e^-_\downarrow$ go $L \;\to\; 0$:

Spin current.

But no electric current.



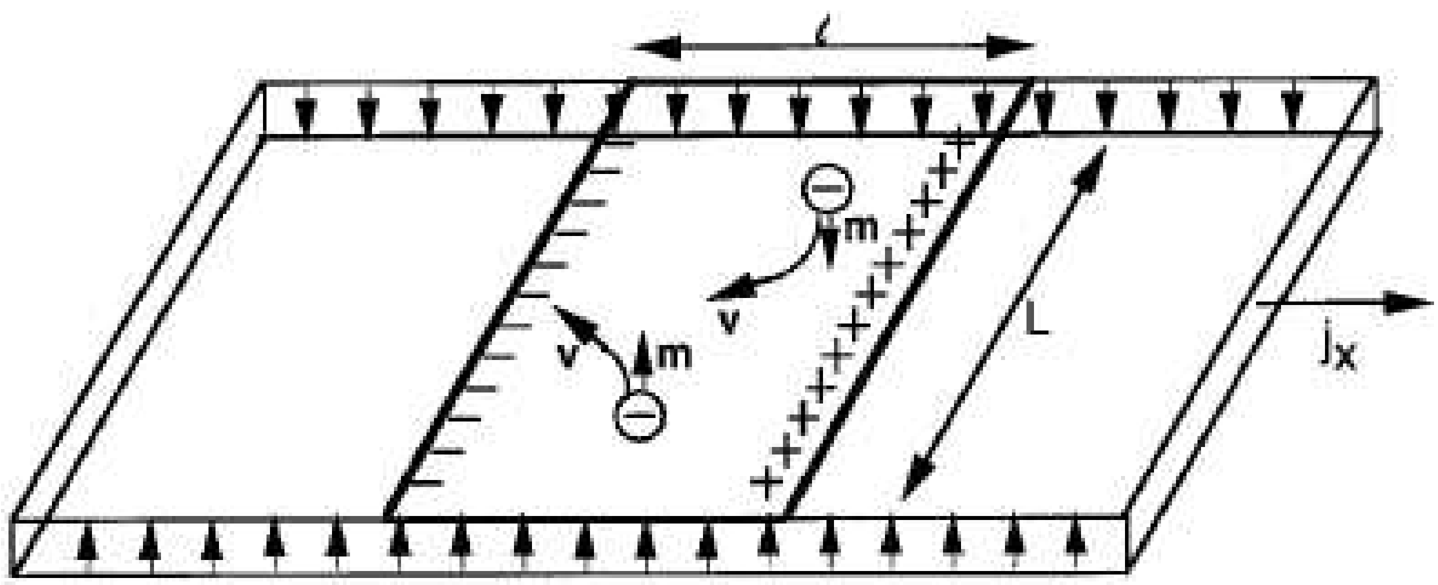
How to detect

Connect $y = 0$ and $y = L$ with another slab of the same material:

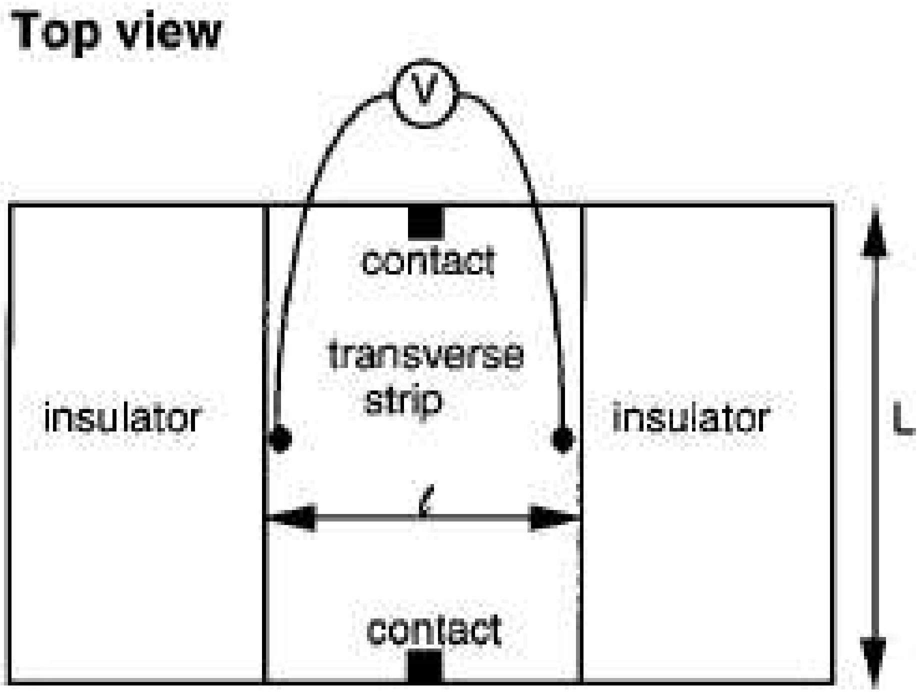
$e^-_\uparrow$ go $0 \rightarrow L$ and scatter to their left.

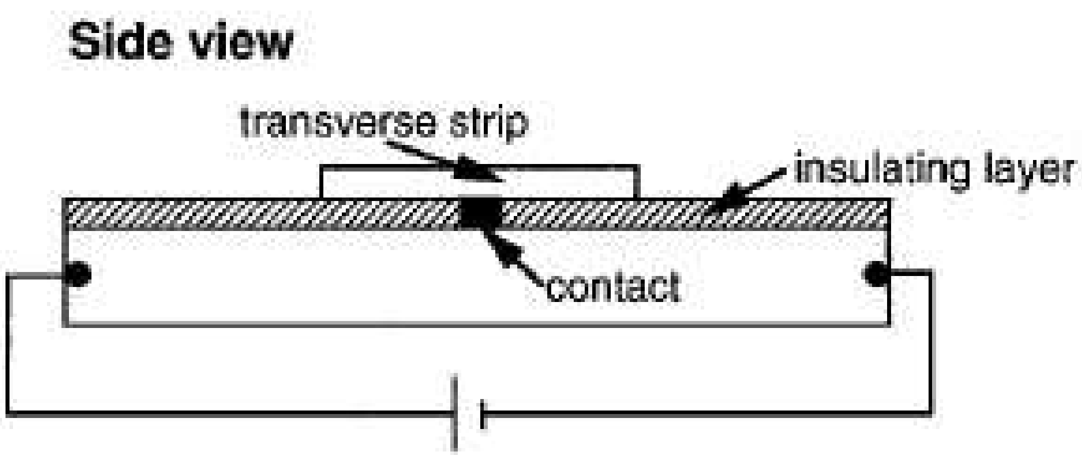
$e^-_\downarrow$ go $L \rightarrow 0$ and scatter to their right.

Voltage between left and right edges of the small slab!



How to detect





Math

Consider only $e^-_\uparrow$'s. "Magnetization" due to $e^-_\uparrow$: $M = n_\uparrow \mu_B$

Anomalous Hall voltage:

$V_H = 4\pi R_s L j_x n_\uparrow \mu_B$.

$V_{SH} = V_H = 2\pi R_s L j_x n \mu_B$.

Small slab: current for each spin

$j_\sigma = \frac{V_{SH}}{\rho L} = \frac{2\pi R_s n \mu_B}{\rho}$

Resulting Spin Hall voltage $V^\sigma_{SH} = 4\pi R_s l j_\sigma n_\sigma \mu_B$

Add voltages from both spins $\qquad V_{SC} = 8\pi^2 R_s^2 l \frac{(n\mu_B)^2}{\rho} j_x$

For different materials $\qquad V_{SC} = 8\pi^2 R_{s1} R_{s2} l \frac{(n\mu_B)^2}{\rho_2} j_x$

Quantum Hall Effect

Ordinary Hall Effect in 2-D:

$\vec{B} \parallel \hat{z}$

Resistivity tensor

$$\rho = \begin{bmatrix} \rho_0 & \rho_H \\ -\rho_H & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & B/ne \\ -B/ne & \rho_0 \end{bmatrix}$$

Quantum Hall Effect

Quantum Hall Effect in 2-D:

$\vec{B} \parallel \hat{z}$

Resistivity tensor

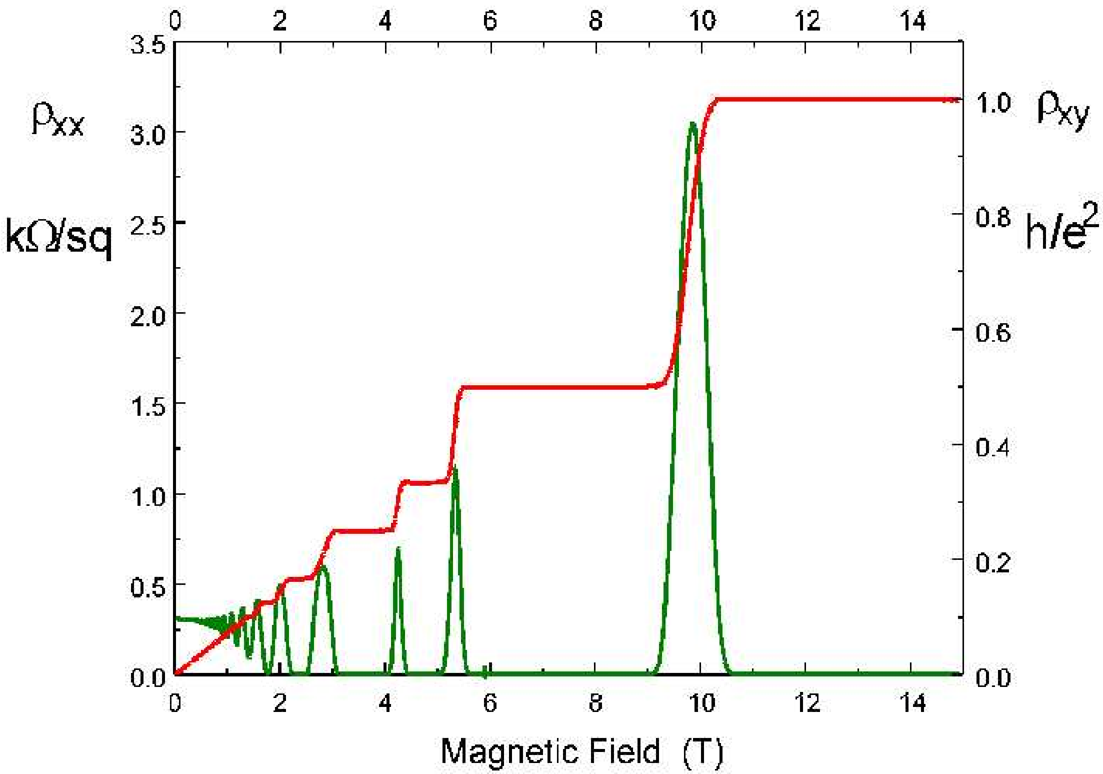
$$\rho = \begin{bmatrix} \rho_0 & \rho_H \\ -\rho_H & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & h/ie^2 \\ -h/ie^2 & \rho_0 \end{bmatrix}$$

i - an integer.

$\rho_H = h/ie^2 \quad \equiv \quad \sigma_H = ie^2/h$ - quantized.

Quantum Hall Effect

Integer quantum Hall effect in a GaAs-GaAlAs heterojunction, T =30mK. Also diagonal component of resistivity - shows regions of zero resistance corresponding to each QHE plateau.



A Derivation

A plane: $0 < x < L$, $0 < y < W$.

Landau gauge: $A_x = -yB,\ A_y = 0$.

S.E.:

$$\frac{\hbar^2}{2m}\left[\left(-i\frac{\partial}{\partial x} - \frac{eB}{\hbar}y\right)^2 - \frac{\partial^2}{\partial y^2}\right]\psi = E\psi$$

Trying $\psi = e^{ikx}\phi(y)$:

$$\frac{h\omega_c}{2}\left[-l^2\frac{\partial^2}{\partial y^2} + \left(\frac{y}{l} - lk\right)^2\right]\phi = E\phi$$

where $l \equiv\ (\hbar/eB)^{1/2}$.

- H.O. centered at $y = l^2 k \Rightarrow\ 0 < k < W/l^2$.

Solutions are $\phi_{nk}(y) = H_n(y/l - lk)e^{-(y-l^2k)^2/2l^2}$

$E_{nk} = \hbar\omega_c(n + \frac{1}{2})$ - independent of k.
Periodic B.C. on x: $x = 0 \equiv x = L \Rightarrow k = 2\pi p/L$
$\Rightarrow$ number of states in Landau level $LW/2\pi l^2$, or per unit area $n_B = 1/2\pi l^2 = eB/h$.
If every occupied level is full, then "filling factor"

$$\nu = \frac{n}{n_B}$$

- integer, or

$$n = \nu n_B = \nu \frac{eB}{h}$$

$$\rho_H = \frac{B}{ne} = \frac{h}{\nu e^2}$$