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Introduction to Spin Hall Effect

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Outline

- Hall effect
- Anomalous Hall effect
- Spin Hall effect from Anomalous Hall effect
- Detection of Spin Hall effect
- Quantum Hall effect
Hall effect

Electric current in magnetic field $\vec{B} \perp \vec{j}$

$\Rightarrow \vec{E}_H \perp \vec{j}, \vec{B}$

$E_H = \rho_H j$

$\rho_H = R_0 B$

$R_0$ - Hall coefficient. Easy to show: $R_0 = \frac{1}{en}$
Math

Electron moving in $\vec{B} \perp \vec{v}$.
Equilibrium between the forces:

\[ evB = eE \]
\[ j = nev \]
\[ \Rightarrow E = j \frac{1}{ne} B \]
\[ \rho_H = \frac{E}{j} = \frac{1}{ne} B \]
Resistivity tensor (2D, $\vec{B} \parallel \hat{z}$):

$$
\rho = \begin{bmatrix}
\rho_0 & \rho_H \\
-\rho_H & \rho_0
\end{bmatrix} = \begin{bmatrix}
\rho_0 & B/ne \\
-B/ne & \rho_0
\end{bmatrix}
$$
Anomalous Hall effect

Electric current in magnetic field $\vec{B} \perp \vec{j}$

$\Rightarrow \vec{E}_H \perp \vec{j}, \vec{B}$

$E_H = \rho_H j$

$\rho_H = R_0 B + 4\pi R_s M$

$M$ - magnetization

$R_s$ - anomalous Hall coefficient

Origin: A few mechanisms proposed: skew scattering by impurities and phonons, ”side jump” mechanism, other.

Exists in FM $\Rightarrow e^-$’s carrying spin and associated magnetic moment experience transverse force to their movement.
Spin Hall effect

No magnetization - e.g. paramagnet.
But scattering mechanisms causing Anomalous Hall effect exist:
$e_{\uparrow}^-$ scatter preferentially in one direction, $e_{\downarrow}^-$ in another.
How to detect?

Hall effect:
Connect $y = 0$ and $y = L$:
$e^-$ go 0 $\to$ L:
Electric current.

Spin Hall effect:
Connect $y = 0$ and $y = L$:
$e^-$ $\uparrow$ go 0 $\to$ L
$e^-$ $\downarrow$ go L $\to$ 0:
Spin current.
But no electric current.
How to detect

Connect $y = 0$ and $y = L$ with another slab of the same material:

$e_{\uparrow}^-$ go $0 \rightarrow L$ and scatter to their left.
$e_{\downarrow}^-$ go $L \rightarrow 0$ and scatter to their right.

Voltage between left and right edges of the small slab!
How to detect
Consider only $e^-$’s. ”Magnetization” due to $e^-$: $M = n_{\uparrow} \mu_B$

Anomalous Hall voltage:

$V_H = 4\pi R_s L j_x n_{\uparrow} \mu_B$.

$V_{SH} = V_H = 2\pi R_s L j_x n_{\uparrow} \mu_B$.

Small slab: current for each spin

$j_\sigma = \frac{V_{SH}}{\rho L} = \frac{2\pi R_s n_{\mu_B}}{\rho}$

Resulting Spin Hall voltage $V_{SH}^\sigma = 4\pi R_s l j_\sigma n_\sigma \mu_B$

Add voltages from both spins

$V_{SC} = 8\pi^2 R_s^2 l \frac{(n_{\mu_B})^2}{\rho} j_x$

For different materials

$V_{SC} = 8\pi^2 R_{s1} R_{s2} l \frac{(n_{\mu_B})^2}{\rho_2} j_x$
Quantum Hall Effect

Ordinary Hall Effect in 2-D: 
\( \vec{B} \parallel \hat{z} \)
Resistivity tensor

\[
\rho = \begin{bmatrix}
\rho_0 & \rho_H \\
-\rho_H & \rho_0 \\
\end{bmatrix} = \begin{bmatrix}
\rho_0 & B/ne \\
-B/ne & \rho_0 \\
\end{bmatrix}
\]
Quantum Hall Effect

Quantum Hall Effect in 2-D:
\[ \vec{B} \parallel \hat{z} \]
Resistivity tensor

\[
\rho = \begin{bmatrix}
\rho_0 & \rho_H \\
-\rho_H & \rho_0
\end{bmatrix}
= \begin{bmatrix}
\rho_0 & \hbar/ie^2 \\
-h/ie^2 & \rho_0
\end{bmatrix}
\]

\(i\) - an integer.

\(\rho_H = \hbar/ie^2 \quad \equiv \quad \sigma_H = ie^2/\hbar\) - quantized.
Quantum Hall Effect

Integer quantum Hall effect in a GaAs-GaAlAs heterojunction, $T = 30 \text{mK}$. Also diagonal component of resistivity shows regions of zero resistance corresponding to each QHE plateau.
A Derivation

A plane: $0 < x < L$, $0 < y < W$.
Landau gauge: $A_x = -yB$, $A_y = 0$.
S.E.:

$$\frac{\hbar^2}{2m} \left[ \left( -i \frac{\partial}{\partial x} - \frac{eB}{\hbar} y \right)^2 - \frac{\partial^2}{\partial y^2} \right] \psi = E\psi$$

Trying $\psi = e^{ikx} \phi(y)$:

$$\frac{\hbar\omega_c}{2} \left[ -l^2 \frac{\partial^2}{\partial y^2} + \left( \frac{y}{l} - lk \right)^2 \right] \phi = E\phi$$

where $l \equiv (\hbar/eB)^{1/2}$.
- H.O. centered at $y = l^2k \Rightarrow 0 < k < W/l^2$.
Solutions are $\phi_{nk}(y) = H_n(y/l - lk)e^{-(y-l^2k)^2/2l^2}$
\[ E_{nk} = \hbar \omega_c (n + \frac{1}{2}) \text{ - independent of } k. \]

Periodic B.C. on \( x \): \( x = 0 \equiv x = L \Rightarrow k = \frac{2\pi p}{L} \)

\( \Rightarrow \) number of states in Landau level \( LW/2\pi l^2 \), or per unit area \( n_B = 1/2\pi l^2 = eB/\hbar \).

If every occupied level is full, then "filling factor"

\[
\nu = \frac{n}{n_B}
\]

- integer, or

\[
n = \nu n_B = \nu \frac{eB}{\hbar}
\]

\[ \rho_H = \frac{B}{ne} = \frac{\hbar}{\nu e^2} \]