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Quantum Computing with Optical Lattices talk 5-11-2007

Quantum Computing with Optical Lattices
• What is Quantum Computing
• Requirements for quantum computing system
• What is Optical lattice
• Quantum computing with optical lattices
• Entanglement of atoms by cold controlled collisions
What is Quantum Computing

Classical computer with 3 bit register:
Bits in register are in a definite state, eg. 101.
Quantum computer:
Qubits are in a superposition

\[ |\psi\rangle = a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle \]

a, b, c,... - complex numbers
\[ |a|^2, |b|^2, |c|^2 \ldots \] - the probability to measure the register in the state \(|000\rangle, |001\rangle, |010\rangle\)
For \(n\) qubit register, recording its state requires \(2^n\) complex numbers.
Initialization, execution and termination
In our example, the contents of the qubit registers can be thought of as an 8-D complex vector.
An algorithm:
Initialize this vector in some specified form (dependent on the design of the quantum computer).
\[ |\psi\rangle = |\psi_0\rangle \]
In each step of the algorithm, that vector is modified by multiplying it by a unitary matrix.
\[ |\psi_n\rangle = U_n |\psi_{n-1}\rangle \]
The matrix is determined by the physics of the device. The unitary character of the matrix ensures the matrix is invertible (so each step is reversible).
Termination of the algorithm: the 8-D complex vector stored in the register. Measurement: will yield a random 3 bit string (and will destroy the stored state). This random string can be used in computing the value of a function because (by design) the probability distribution of the measured output bitstring is skewed in favor of the correct value of the function. By repeated runs of the quantum computer and measurement of the output, the correct value can be determined, to a high probability, by majority polling of the outputs.
A quantum algorithm is implemented by an appropriate sequence of unitary operations. Note that for a given algorithm, the operations will always be done in
exactly the same order. There is no ”IF THEN” statement to vary the order, since there is no way to read the state of a qubit before the final measurement.
• Requirements for quantum computing system

1. A scalable physical system with well characterized qubits.

2. The ability to initialize the state of the qubits to a simple fiducial state, such as $|000...\rangle$.

3. Long relevant decoherence times, much longer than the gate operation time.


5. Qubit-specific measurement capability.

Many of these requirements are fulfilled for atoms in optical lattices
• What is Optical lattice

- Formed by counterpropagating laser beams.
Resulting periodic potential used to trap neutral atoms.
Trapped atoms resemble a crystal: localized periodically in space.
• Quantum computing with optical lattices

• Many of the requirements for quantum computing system satisfied:

• Scalable and well defined set of qubits with long decoherence times.

• Natural Cubit: 2 metastable ground states of atom $|a\rangle$ and $|b\rangle$.

• Initialization: Setting MI state with one particle/site.

• Challenge: construct quantum gates between atoms trapped on different sites. 1999 Dieter Jaksch, Ignacio Cirac, Peter Zoller and co-workers: State-dependent optical potentials used to bring neighbouring atoms together on a single lattice site.
• Entanglement of atoms by cold controlled collisions

Consider 2 atoms in internal states $|a\rangle$ and $|b\rangle$, in ground states $\psi_0^{a,b}$ in potential wells $V^{a,b}$.
Initially wells centered at $\bar{x}^{a,b}$, far apart, atoms not interacting.
Move wells $\bar{x}^{a,b}(t)$, so that wave packets of atoms overlap, and return to the original positions.
\[ H = \sum_{\beta=a,b} \left[ \frac{(p^\beta)^2}{2m} + V^\beta(x^\beta - \bar{x}^\beta(t)) \right] + V_{int}^{ab}(x^a - x^b) \]

Adiabatic limit: Wave function accumulates phase

\[ \psi_0^a(x^a - \bar{x}^a)\psi_0^b(x^b - \bar{x}^b) \rightarrow e^{i\phi} \psi_0^a(x^a - \bar{x}^a)\psi_0^b(x^b - \bar{x}^b) \]

\[ \phi = \phi^a + \phi^b + \phi^{ab} \]

\( \phi^a, \phi^b \) - kinematic phases;

\( \phi^{ab} \) - interaction (collision) phase

\[ \phi^{ab} = \frac{1}{\hbar} \int dt \Delta E(t) \]

Maximal entanglement - by collision with phase \( \phi^{ab} = \pi \).

Two states: use two spins. To create spin-dependent potentials: use two counter-propagating lasers with polarizations that can be rotated relative to one another in a few microseconds. By exploiting this control over the beams, both internal states of the atom can experience different lattice potentials such that the atoms can be moved relative to one another.
(a) Spin-dependent potential used for Controlled interactions between atoms on different lattice sites. These lattices moved relative to each other such that two initially separated atoms can be brought into controlled contact.

(b) This can be extended to form a massive parallel quantum-gate array. Consider a string of atoms on different lattice sites. First the atoms are placed in a coherent superposition of the two internal states (represented by red and blue). Spin-dependent potentials are then used to ”split” each atom such that it simultaneously moves to the right and to the left, and is brought into contact with the neighbouring atoms. Here, both atoms interact and a controlled phase shift, $\phi$, is introduced between them. After such a controlled collision the atoms are again moved back to their original lattice sites.
• Controlling lattice shifts - possible to move the atoms over a precisely defined separation and to bring them into contact with very distant neighbouring atoms.

• High degree of control over atoms in an optical lattice. We can completely control the interaction between two atoms

• Periodic potential ⇒ powerful "parallelism": single lattice-shift operation can bring each atom into contact with its neighbouring atom, forming a powerful quantum-gate array. Quantum-gate array ⇒ generate entangled many-body states.
A challenge: address single atoms on different lattice sites. ⇒ read and write information into selected atoms. Some success - Dieter Meschede et al, University of Bonn, Germany: Use magnetic fields to select single atoms that are separated by a few lattice sites.
References


