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CLOSED TIMELIKE CURVES

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Abstract. In this overview article, we studying the possibility of closed timelike curves (CTC), or "time travel". We consider how CTC's appear in Special and general theory of relativity, nd study a few examples of solutions to the equations of GR, leading to CTC's. We also briefly touch related topics - CTC's in microworld, and Many Worlds Interpretation of quantum mechanics.

Outline of the paper

This paper investigates the question: Are closed timelike curves (CTC) possible?

1. We give a brief history of the problem of CTC. A few solutions to the Einstein's field equations make us conclude that CTC's are possible in general relativity.

2. We consider the main objection to the possibility of the existence of CTC's: chronology protection conjecture, with conclusion that it is a serious argument, but not yet conclusive.

3. We given brief accounts of consistency argument, many worlds interpretation, microworld CTC's. These topics are closely related to the subject of the paper and deserve mentioning, but are too large in themselves to give any detailed account here.

1. History

Closed timelike curves in Special Theory of Relativity. There are no CTC's in flat spacetime. Lorentz transformations make coordinates of space and time relative. The sequence of events may also be relative, but not when points lie on a timelike curve: there, the sequence of events is the same for all observers.

Closed timelike curves in General Relativity. In curved spacetime, it becomes more interesting. Locally - in infinitesimally small region - curves still have to behave respectfully to chronology. But globally, turning and twisting with manifold, the curve may end intersecting itself. Ingenious physicist "only" have to think out a manifold which will allow CTC, and matching stress-energy distribution.

1.1. **Gödel's Universe.** Probably the first such solution was found, or invented, by Gödel in 1949. His "An Example of a New Type of Cosmological Solutions of Einstein's Field Equations of Gravitation" [1] presents a universe of rotating matter. In his paper Gödel stated a few interesting properties of his universe. Th most interesting property of his universe was the fact that it had closed timelike curves passing through every point.

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Gödel's metric (using convention of -2 signature):

$$ds^2 = a^2 \left[dx_0^2 - dx_1^2 + (e^{2x_1}/2)dx_2^2 - dx_3^2 + 2e^{x_1}dx_0dx_2\right]$$

The field equation (for dust):

$$R_{ik} - \frac{1}{2}Rg_{ik} = 8\pi\kappa\rho u_i u_k + \Lambda g_{ik}$$

We are primarily interested in the metric, so we will skip the derivation for the matter. The important point is that the solution exists. Gödel's solution is

$$\frac{1}{a^2} = 8\pi\kappa\rho, \quad \Lambda = -\frac{R}{2} = -\frac{1}{2a^2} = -4\pi\kappa\rho$$

Now the metric. Changing to new coordinates (r, ϕ, t, y):

$$e^{x_1} = \cosh(2r) + \cos\phi\sinh(2r),$$

$$x_2 e^{x_1} = \sqrt{2}\sin\phi\sinh(2r),$$

$$\tan\left(\frac{\phi}{2} + \frac{x - 2t}{2\sqrt{2}}\right) = e^{-2r}\tan\frac{\phi}{2}, \ \ where \left|\frac{x_0 - 2t}{2\sqrt{2}}\right| < \frac{\pi}{2}$$

$$x_3 = 2y,$$

the metric becomes

$$ds^2 = 4a^2(dt^2 - dr^2 - dy^2 + (\sinh^4 r - \sinh^2 r)d\phi^2 + 2\sqrt{2}\sinh^2 r d\phi dt)$$

At R big enough, $(\sinh^4 R - \sinh^2 R) > 0$, and so the curve

$$r = R, \ \ y = y_0 = const, \ \ t = t_0 = const, \ \ 0 \le \phi \le 2\pi$$

is a timelike curve. It ends where it started, so it is a closed timelike curve.

While it doesn't seem likely that our universe is Gödel's universe, the very existence of such solution is interesting. Einstein was worried when he heard of this solution.

Since then, many new solutions were found.

1.2. **Frank J. Tipler: Rotating cylinders.** Frank J. Tipler: Rotating cylinders and the possibility of global causality violation.[2]
In 1936 van Stockum solved a problem of infinite rotating cylinder, in which centrifugal forces are balanced by gravitational attraction. The metric:

$$ds^2 = H(dr^2 + dz^2) + Ld\phi^2 + 2Md\phi dt - Fdt^2$$

where $FL + M^2 = r^2$. The solution for the interior:

$$H = e^{-a^2r^2}, \ \ L = r^2(1 - a^2r^2), \ \ \rho = 4a^2e^{a^2r^2}, \ \ M = ar^2, \ \ F = 1,$$

where $a =$ angular velocity of the cylinder. For $r > 1/a$ the lines

$$r = const, \ \ z = const, \ \ t = const, \ \ 0 \le \phi \le 2\pi$$

are CTC's. However, at $r = 1/a$ the velocity of the cylinder must be $ar = 1 \equiv c$, so maybe there is no causality violation. Assuming the radius of a cylinder $1/2a < R < 1/a$, the solution is

$$H = e^{-a^2R^2}(\gamma/R)^{-2a^2R^2}, \ \ L = \frac{R\gamma\sin(3\beta+\gamma)}{2\sin 2\beta\cos\beta}, \ \ M = \frac{\gamma\sin(\beta+\gamma)}{\sin 2\beta}, \ \ F = \frac{\gamma\sin(\beta-\gamma)}{R\sin\beta},$$

where $\gamma = (4a^2R^2 - 1)^{1/2}\ln(\gamma/R), \ \ \beta = \tan^{-1}(4a^2R^2 - 1)^{1/2}$. With sinusoides in the expression for L, L can be negative, and again the lines

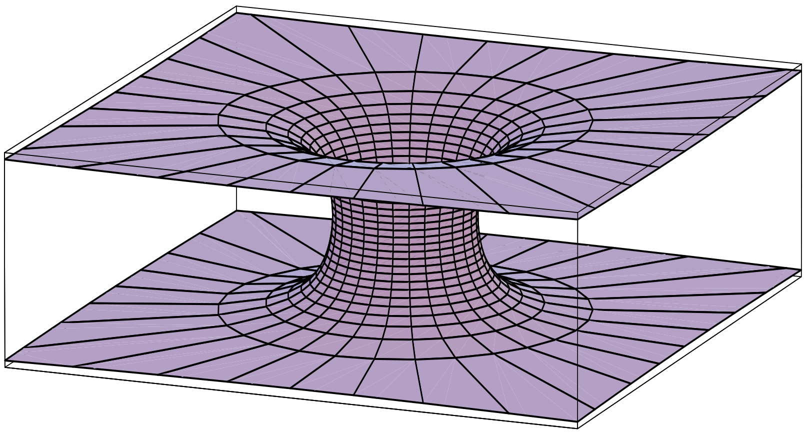
$$r = const, \ \ z = const, \ \ t = const, \ \ 0 \le \phi \le 2\pi$$

are CTC's.

1.3. **Thorne et al: Wormholes.** Morris, Thorne and Yurtsever describe in "Wormholes, Time Machines, and the Weak Energy Condition" [3] a way of using wormholes for creating Closed Timelike Curves.

A popular account is given in "Wormholes in spacetime and their use for interstellar travel: A tool for teaching general relativity" [4]

What makes wormholes useful is not particulars of their metrics, but the fact that the path through the wormhole from one mouth to another is very short comparing to the path through the surrounding spacetime.



Suppose we have a wormhole with mouthes A and B. At the beginning, their time is synchonized in our system. Now let's accelerate the wormhole mouth B to nearly the speed of light for some time, and then bring them together. Let's say in "motionless" system the time passed was T, and in accelerated system, moving with the wormhole mouth B, the time passed was T'. Then we have a time machine for travelling $\Delta T = (T - T')$ to the past: If we dive into the wormhole mouth B at time t, we'll emerge from the wormlole mouth A at time $t - \Delta T$.

The possibility of wormholes as Schwarzschild solution to the Einstein's field equations first was suggested by Ludwig Flamm in 1916.

However, there are serious objections to the existence and usability of Schwarzschild wormholes:

• Tidal forces of such wormhole must be very large - about the same magnitude as at the horizon of a Schwarzschild black hole. They may be bearable though if wormhole's mass is $\sim\ 10^4$ solar mass;

• Schwarzschild wormhole isn't static, but dynamic: it expands from zero to maximum throat circumference, and then back to zero - so fast that it's not possible to pass through the wormhole even moving at the speed of light;

• Schwarzschild wormhole has a past event horizon which is unstable against small perturbations.

Thorne et al found a different class of solutions for field equations than Schwarzschild wormholes they call their solutions

Traversible wormholes.

An example of a metric for a wormhole:

$$ds^2 = -dt^2 + dl^2 + (b_0^2 + l^2)(d\theta^2 + \sin^2\theta d\phi^2),$$

where $-\infty < t < +\infty,\ -\infty < l < +\infty,\ 0 \leq \theta \leq \pi,\ 0 \leq \phi \leq 2\pi,\ b_0$=const.

It is easy to see that such metric describes 3-D bottleneck, or handle, connecting two asymptotically flat regions of spacetime ($l \to -\infty$ and $l \to +\infty$) , or two universes.

It may be shown that the tidal forces for an object moving through the wormhole $\to 0$ if $v \to 0$, i.e. they may be made arbitrarily small for slow-moving object. This would make wormhole traversable.

However, there may be an obstacle: calculating stress-energy tensor gives

$$-T^{tt} = -T^{ll} = T^{\theta\theta} = T^{\phi\phi} = \frac{1}{8\pi G}\frac{b_0^2}{(b_0^2 + l^2)^2}$$

- negative energy density, or "exotic matter".

Thorne and Morris farther proceed to derive a better condition than this: they find that at the throat the conditions must be

$$\tau_0 > \rho_0 > 0,$$

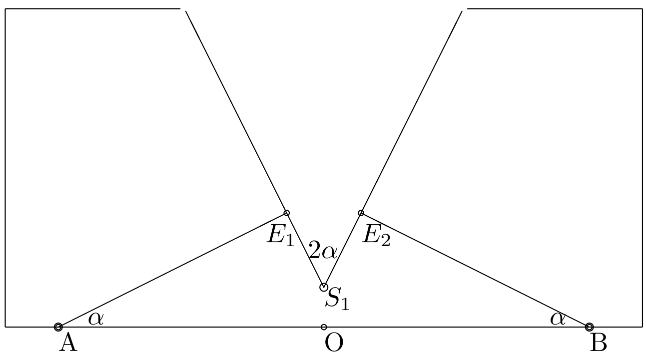
tension at the throat must be greater than mass-energy density. It is not good enough and will give negative energy density for a relativistically moving observer

$$\rho_0' = \gamma^2(\rho_0 - \tau_0) + \tau_0,$$

which will be negative when γ gets large enough with increase of velocity. So it seems that Thorne will need a material with negative energy density to construct his wormhole. But so far we did not find negative energy, outside of Casimir effect. This may be a major obstacle to the possibility of building a traversible wormhole.

1.4. **J. Richard Gott III: Cosmic Strings.** J. Richard Gott III: Closed timelike curves produced by pairs of moving cosmic strings: Exact solutions [5]

Cosmic strings make interesting geometry - they cut out of space a sector, so that a full circle around such string is less than 2π. Otherwise, spacetime is left flat.



Let us consider a cosmic string S_1 - in 2-D cross-section.

Take point O at a distance d from it.

Draw a straight line (a geodesic) $AOB \perp S_1O$; $AO = OB = x_0$.

There is another geodesic line passing through A and B on the other side of S_1. With mapping on 2-D Cartesian surface, denote this other geodesic AE_1E_2B.

Since S_1 cuts out of a plane a wedge of an angle 2α, the angles $\angle OAE_1 = \angle OBE_2 = \alpha$.

Then, if we take point A far enough from O, the distance AE_1 will be less than AO. (We don't need the exact formula. Just observe that if we move A to infinity, then $AO \approx AS_1$; and $AE_1 \approx AS_1 \cos\alpha$.)

So, if we send from point A two light beams, moving by geodesics AOB and AE_1E_2B, the beam moving by AE_1E_2B will beat the beam moving by AOB.

Therefore, a particle (or a rocket) m_1 moving fast enough by AE_1E_2B will beat the beam moving by AOB.

So, in the frame of AOB these events - departure of a particle at the point A and its arrival at the point B are spacelike. Choosing suitable inertial frame, we can make these events simultaneous.

Now, in this new frame, we have S_1 moving relative to AOB.

Let us symmetrize the picture by adding another cosmic string S_2, symmetric to S_1 relative to the point O. And a particle m_2 moving from point B to point A around the string S_2; due to the symmetry, in the AOB frame, m_2 departs from B and arrives to A simultaneously.

The resulting path of m_1 from A to B and m_2 from B to A (which can just as well be the same particle), makes a CTC.

Of course, in reality the existence of infinite cosmic strings doesn't seem likely.

Still, it is a legitimate solution to the field equations.

Also - the main reason of arising CTC seems to be gravitational lensing, and it is real without such exotic objects as cosmic strings. Can CTC be produced by shooting massive bodies past each other?

Conclusion. General Relativity technically allows CTC's.

Can CTC's exist in the real world?

2. Chronology Protection Conjecture [6, 7]

The main objection to the existence of the CTC's is formulated as chronology protection conjecture.

In this section:

About a Cauchy horizon - which is the important subject of the chronology protection conjecture.

Cosmic censorship hypothesis - a conjecture closely related to the chronology protection conjecture.

Chronology protection conjecture.

The conclusion: Conjecture is a strong argument, but not conclusive.

2.1. Cauchy Horizon.

Cauchy horizon is a light-like boundary of the domain of validity of a Cauchy problem.

In more detail:

Partial Cauchy surface S is defined as a subset of manifold M, such that S is achronal, closed and has no edge.

Future domain of dependence of S $D^+(S)$ - the set of all points p, such that every past-moving inextendible causal curve through p must intersect S.

Future Cauchy horizon $H^+(S)$ - boundary of $D^+(S)$.

Past domain of dependence of S $D^-(S)$ and **past Cauchy horizon** $H^-(S)$ are defined similarly.

2.2. Penrose: Cosmic Censorship Hypothesis [8, 9].

This is related to the Chronology Protection Conjecture - dealing also with causality and Cauchy surfaces.

In a way, it is predecessor of Chronology Protection Conjecture.

Both conjectures hypothesize the existence of "Cosmic Censorship". They can be said to complement each other.

Suppose we have a singularity at a point p.

Let us take some partial Cauchy surface at p's past.

Singularity doesn't belong to the manifold, so if we take a future domain of dependence of S $D^+(S)$, then the singularity will cut from it a cone, thus creating a future Cauchy horizon $H^+(S)$.

The events in this cone depend on singularity. The concern is that, since the physical behavior of singularities is unknown, if singularities can be seen from the rest of spacetime, causality may break down, and physics may lose its predictive power.

Roger Penrose proposed the **Cosmic Censorship Hypothesis** - a conjecture about the nature of singularities in spacetime. The Cosmic Censorship Hypothesis proposes that singularities are always hidden within event horizons, and therefore cannot be seen from the rest of spacetime.

The weak cosmic censorship hypothesis: any observer who has observed a singularity is destined to fall into it.

The strong cosmic censorship hypothesis: no singularity is ever visible to any observer.

2.3. **Hawking: Chronology Protection Conjecture.** Hawking considers creation of a CTC.

Creation of a closed timelike curve necessarily means existence of a Cauchy horizon.

Suppose we start at a region of a nearly flat spacetime, where CTC's don't exist. Let us take some partial Cauchy surface in this region. (Assuming without proof one exists. Can be shown one exists from the fact that CTC's don't exist there.)

Now, suppose we managed to warp spacetime so much that CTC are created.

We did it in a future of S.

By definition, we can't do it to all spacetime - since we started at a region of spacetime where CTC's don't exist.

Let us take a point P_c on some CTC. The point P_c doesn't belong to $D^+(S)$, because at least one timelike path, crossing P_c and leading to the past, doesn't cross S: the CTC itself.

Let us connect P_c to the surface S with some timelike line $L(P_C, P_S)$ (It is an imprecise definition for the line, but enough for the next reasoning). While metric in the region of CTC's did change, and maybe even topology changed, we may be sure that the continuity remains, so that we still should be able to go from one point to another.

On a line $L(P_C, P_S)$, P_S belongs to $D^+(S)$, while P_C doesn't.

Due to the continuity of $L(P_C, P_S)$, there must exist a point on this line P_H such that any neighbourhood of it contains both points belonging to $D^+(S)$, and not belonging to it. I.e. P_H belongs to $H^+(S)$.

All set of all such points P_H - using all points P_C of all CTC's and all points P_S of S, and all possible timelike curves $L(P_C, P_S)$, will comprise $H^+(S)$.

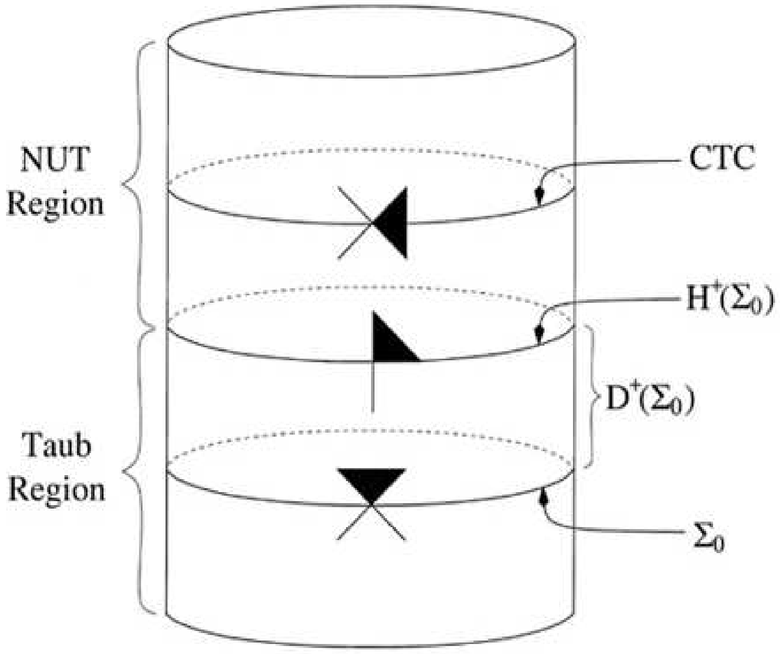
We showed that every timelike curve starting at S and ending at CTC must cross $H^+(S)$.

So, a would-be time traveller, in order to get to a time machine, must cross a Cauchy horizon.

Hawking shows that at the Cauchy horizon the energy-momentum tensor will give metric perturbation that will diverge - which makes the undertaking impossible.

It can be shown that a "finitely generated" Cauchy horizon will contain a closed light ray, a light ray that keeps coming back to the same point over and over again.

It is easy to see on a picture of 1+1 dimensional spacetime:



In a region of CTCs light cones are tilted sideways to allow CTCs;

In a region of no CTCs light cones are pointed upwards;

Between these regions, there is a surface (a line in 1+1 dimensional spacetime) - a Cauchy horizon - where light cones are tilted in such way that they tangentially touch a Cauchy horizon.

A light ray directed along such tangential line will be moving in a Cauchy horizon.

It is evident for 1-dimensional (and finite) Cauchy horizon that this light ray will loop onto itself;

For 3-D (and finite) Cauchy horizon it is somewhat more complicated, but can be shown that some beams will loop onto themselves: Let us take some light beam in a Cauchy horizon, and let it extend to infinity (in length). Consider a 3+2 dimensional space composed of 3-D Cauchy horizon + 2-D of directions (of possible light beams). This 3+2-D space is finite in every direction, and have a finite volume. Our light beam, being infinitely long, but contained in a finite volume, will necessarily have at least one concentration point in 3+2-D space. This concentration point - location + direction - identifies our recurring light beam.

Hawking shows farther that this light ray becomes more and more blueshifted with each loop, i.e. gaining in energy. This would give infinite build up of energy.

Possibly light ray will get defocused enough with each loop, so that not to have an infinite build up of energy.

There's a possibility that the metric perturbation will be cut off by quantum-gravitational effects. Hawking argues however that the metric perturbation will be too large to use this region of closed timelike curves.

Conjecture, not a Law - yet. According to Hawking:

The laws of physics conspire to prevent time travel by macroscopic objects.

So far, it is only a conjecture. Hawking didn't quite prove it. Particularly:

We don't yet know about quantum-gravitational cut-off.

It seems that the possibility remains that some closed tomelike curves existed forever - or since the beginning of the universe.

We don't know yet everything of the large-scale structure of our spacetime. It is usually assumed that the universe is on a large scale uniform and isotropic.

Indeed, the space seems to be uniform and isotropic as far as we can see.

However, we only can see a local region - a sphere of a radius $\sim$ 10 bln. ly.

There is no reason to expect this uniformity to extend to considerably larger distance. And certainly not to infinity, if the universe is infinite.

3. Closed Timelike Curves in Microworld

In Feynman's sum over histories, the particle can take any number of paths between two points, including superluminal, which means time travel is also allowed.

However, the time scale can be only very small, according to the relation

$$\Delta E \Delta t \leq \frac{1}{2}\hbar$$

Closed timelike curves in microworld are quite real and can be observed - indirectly.

Casimir effect can be explained in terms of closed timelike curves: the force between parallel metal plates is caused by the fact that there are less closed-loop histories that can fit between the plates compared with the region outside.

In macroworld: Although closed timelike curves are allowed by the sum over histories, the probabilities are extremely small.

4. Consistency (causality) objections to the possibility of CTC

"Grandfather paradox". A time traveller goes into the past and prevents himself from going into the past.

A rough outline of a solution - here it is assumed that all universe goes through CTC. Suppose we have an initial state of a system $\Psi(t_0)$.

The system develops with time as

$$\Psi(t) = R[t, t_0]\Psi(t_0)$$

R is a "propagator".

If we have a time loop such that $t + T \equiv t$, then we have an equation

$$\Psi(t_0) = R[t_0 + T, t_0]\Psi(t_0)$$

and we need to solve it.

By reasoning of the proponents of "grandfather paradox", we have R such that it will be always

$$\Psi(t_0) \neq R[t_0 + T, t_0]\Psi(t_0)$$

E.g.:

$\Psi(t_0)$ can have only values of 1, -1; and $R \equiv -1\cdot$. Then it follows that

$$\Psi(t_0) = -\Psi(t_0)$$

To argue this:
1. Classically. Ψ can't have a discrete set of values; it must be continuous.
$R[t, t_0]\Psi(t_0)$ must be continuous in t.
Define $y \equiv \Psi(t_0)$, $F(y) \equiv R[t_0 + T, t_0]y$.
We have $F(1) = -1$; $F(-1) = 1$.
Then it's easy to show that equation

$$F(y) = y$$

will always have solution for y, with $-1 < y < 1$. Or

$$\Psi(t_0) = R[t_0 + T, t_0]\Psi(t_0)$$

2. Quantum Mechanically. Change of state is not determined. It will happen only with some probability.

So, the paradox seems to be talked away.
Still, something unusual is happening in the loop - a new kind of constraint. The events seem to be overdetermined.
Moreover, it can be shown that in some way events are also underdetermined. But I'll leave it here.

A good argumentation on this gives Novikov in [7, 12].

5. Many Worlds Interpretation.

First was introduced by Everett as "Relative State" Formulation of Quantum Mechanics [10].

CTC in the light of Many Worlds Interpretation. CTC is not really a closed curve. What were supposed to be the same spacetime coordinates on the loop, after a completed loop, the curve is in another world.
It can be simplistically formalized by introducing another dimension - let us say τ - which is index of "universes". In flat spacetime, τ stays the same. In curved spacetime, τ may change, so in "CTC" the coordinates (t, x, y, z, τ) won't repeat themselves.
In reality τ must be a space of almost infinite number of dimensions.
We can visualize this model in just 3 dimensions: t, x, and τ. The matter of the universe, or rather multiverse, is going as a stream, in general along the t axis, worldlines of particles as lines of stream. The stream in places may bend sideways from the t direction, even go backward in some places. However, the lines of stream don't intersect themselves.

Probably the most known supporter af Many Worlds Interpretation is David Deutsch.

6. Conclusions

• Hawking has some strong arguments to support his Chronology Protection Conjecture.
However, they are not conclusive yet.

• Many Worlds Interpretation, with possible variant of time travel, is interesting.
It isn't developed enough yet, and there is no experimental evidence for this theory.

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