UCDAVIS

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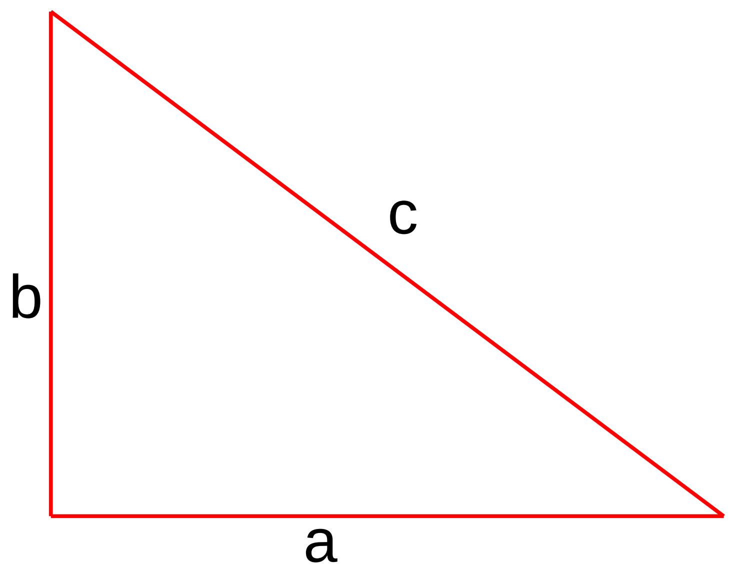
Pythagorean Theorem April 19, 2017

Pythagorean Theorem

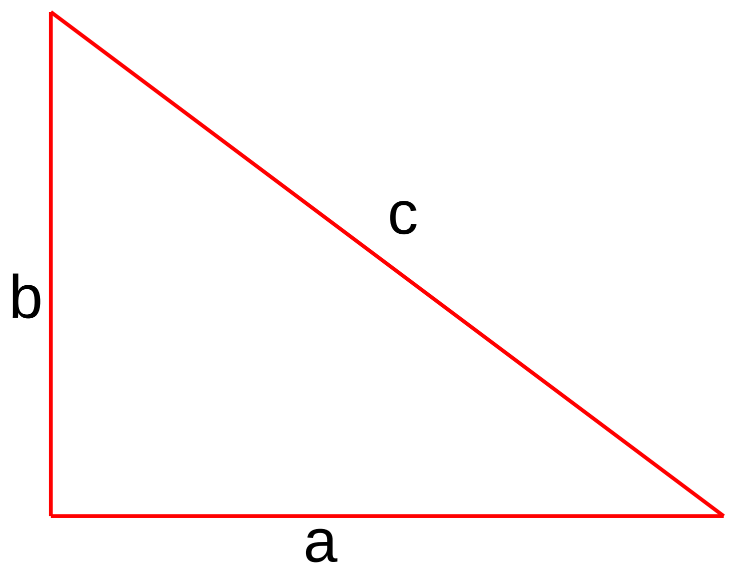
Outline

- Last Time
- Review Quiz
- Statement of Theorem
- Proof
- Exercises
- Applications
- History
- QUIZ

Pythagorean Theorem

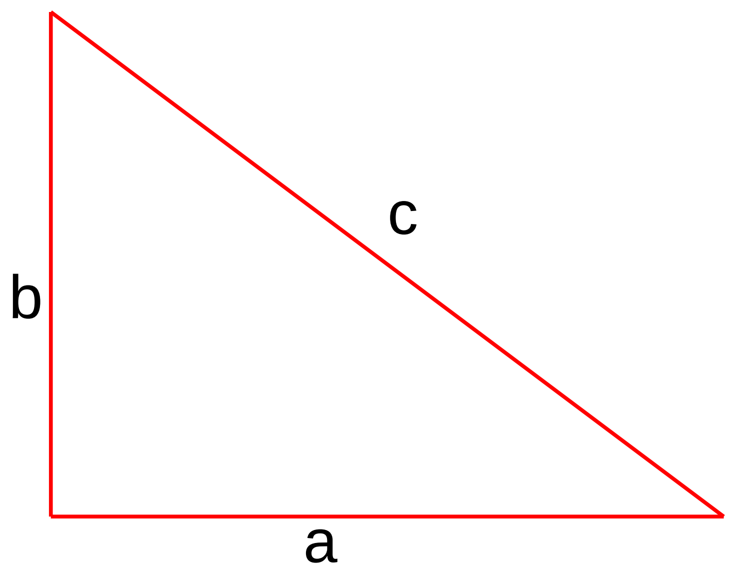


Pythagorean Theorem



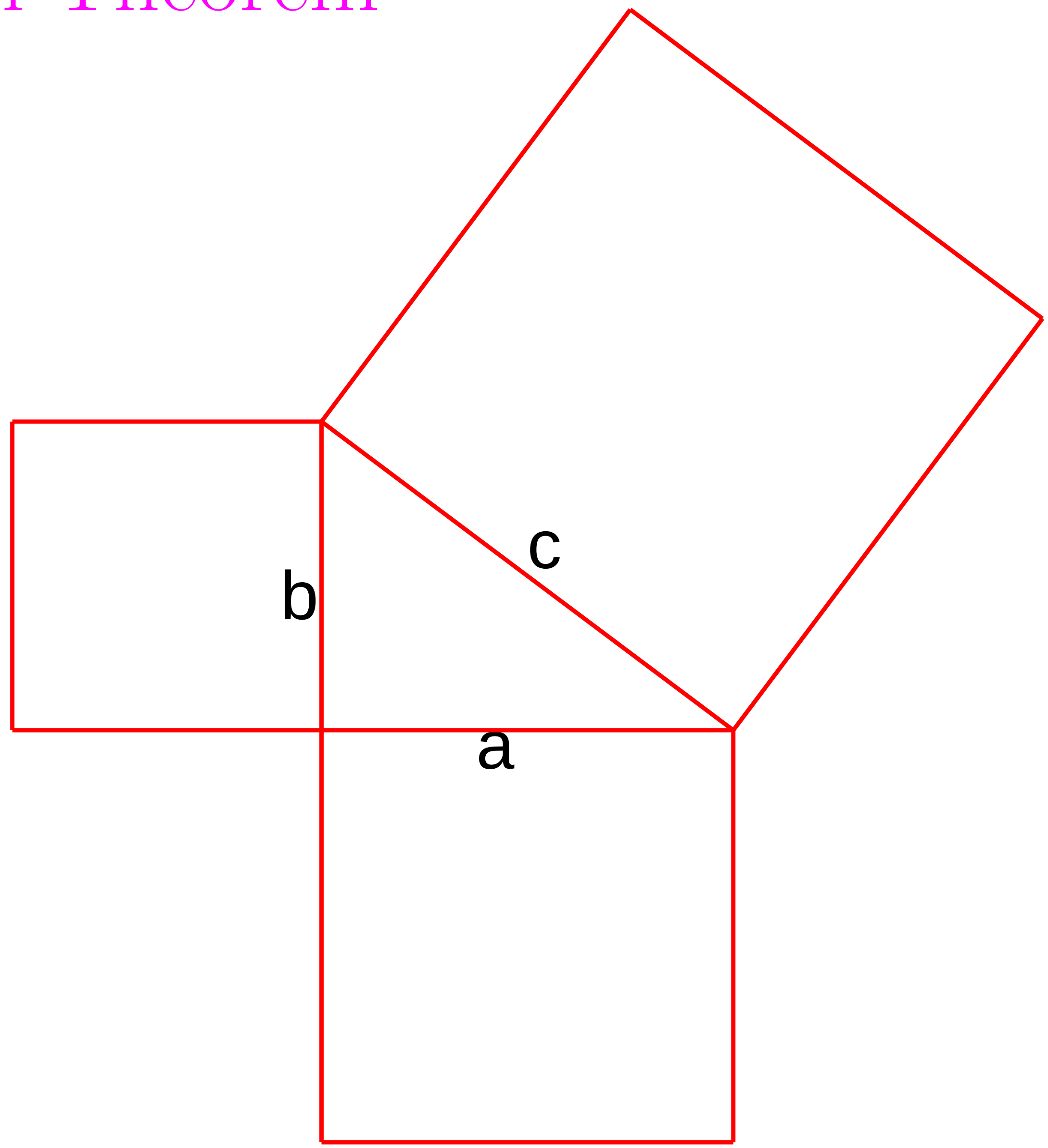
$$a^2 + b^2 = ?$$

Pythagorean Theorem



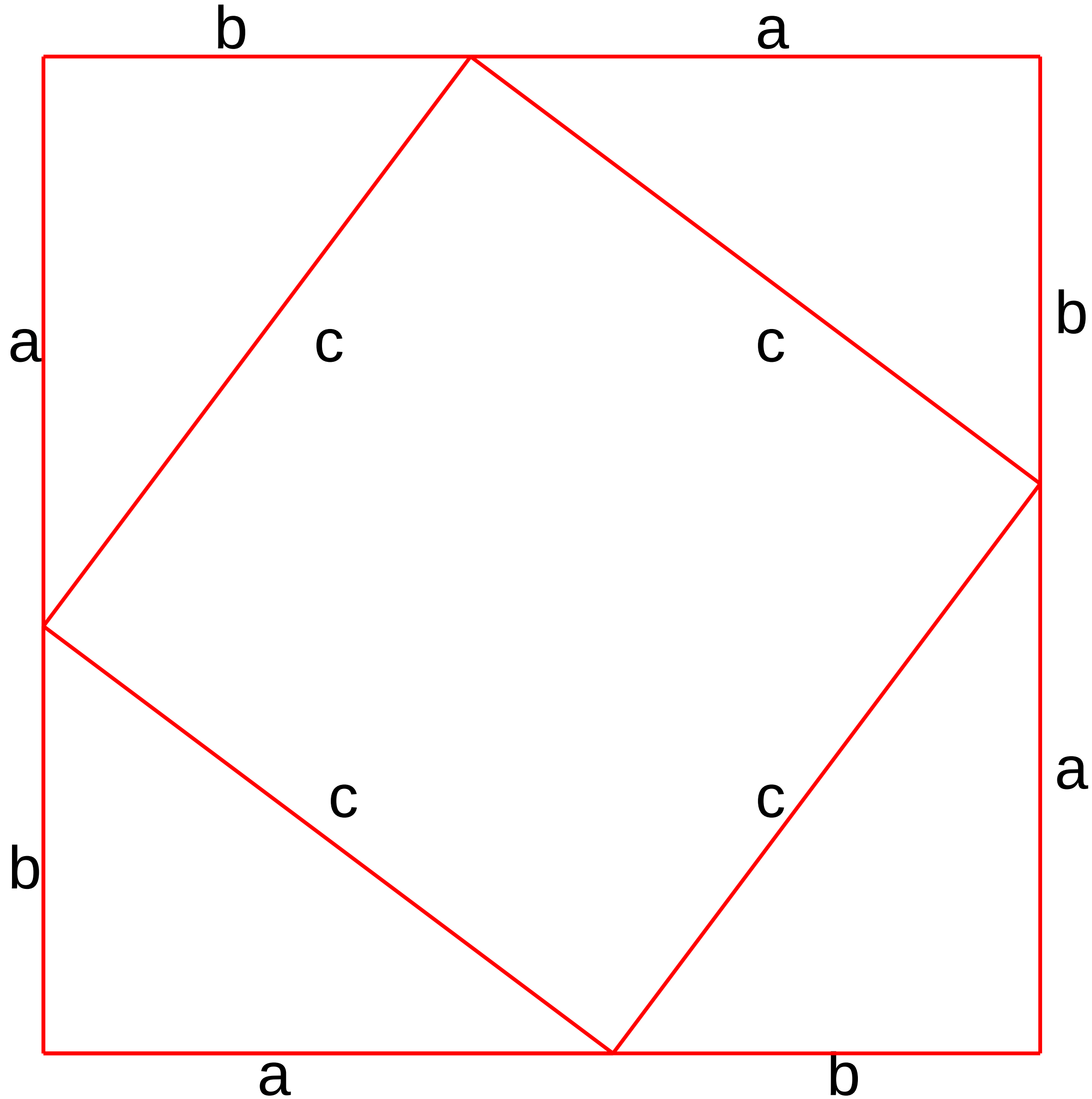
$$a^2 + b^2 = c^2$$

Pythagorean Theorem

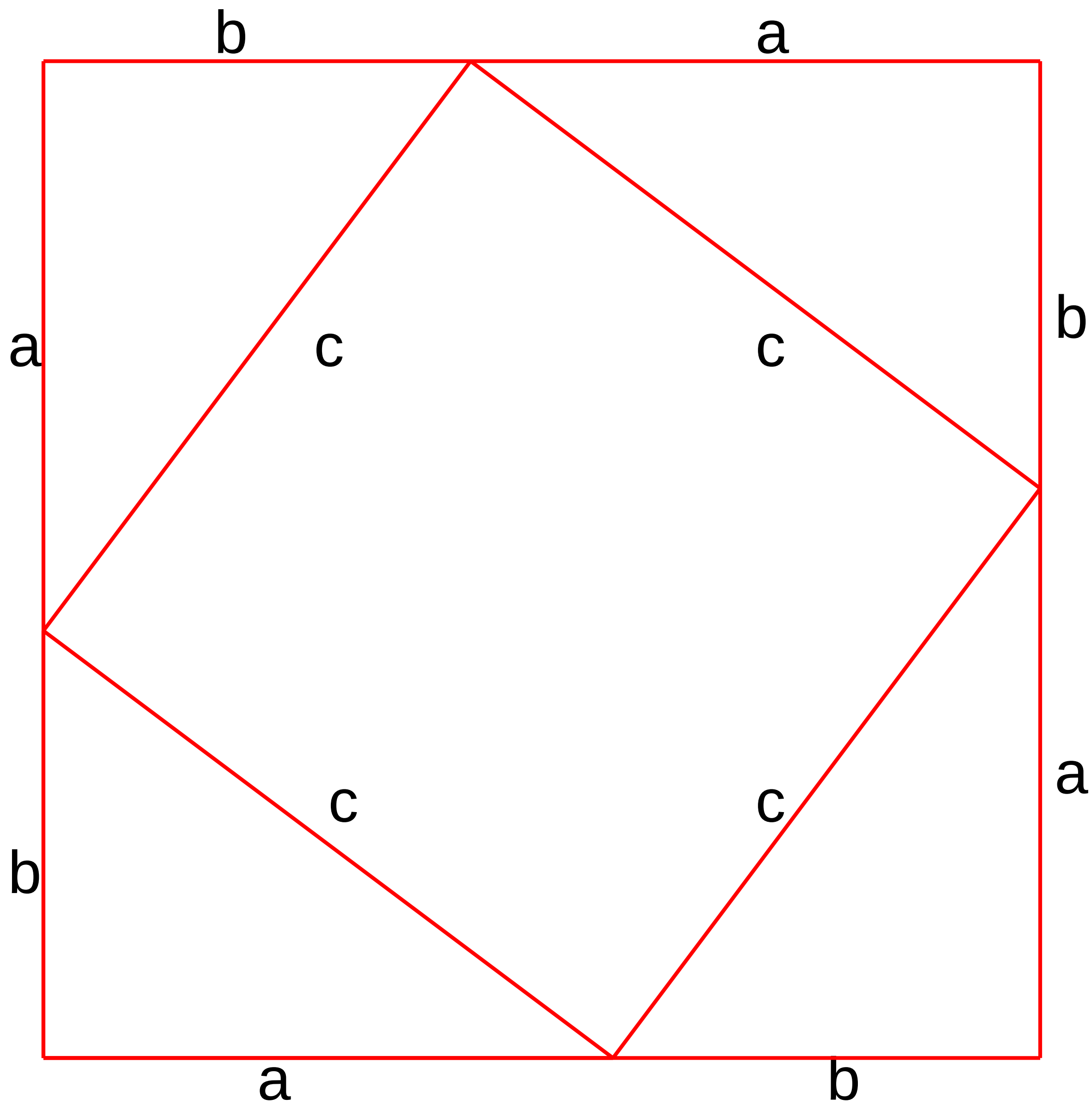


$$a^2 + b^2 = c^2$$

Pythagorean Theorem

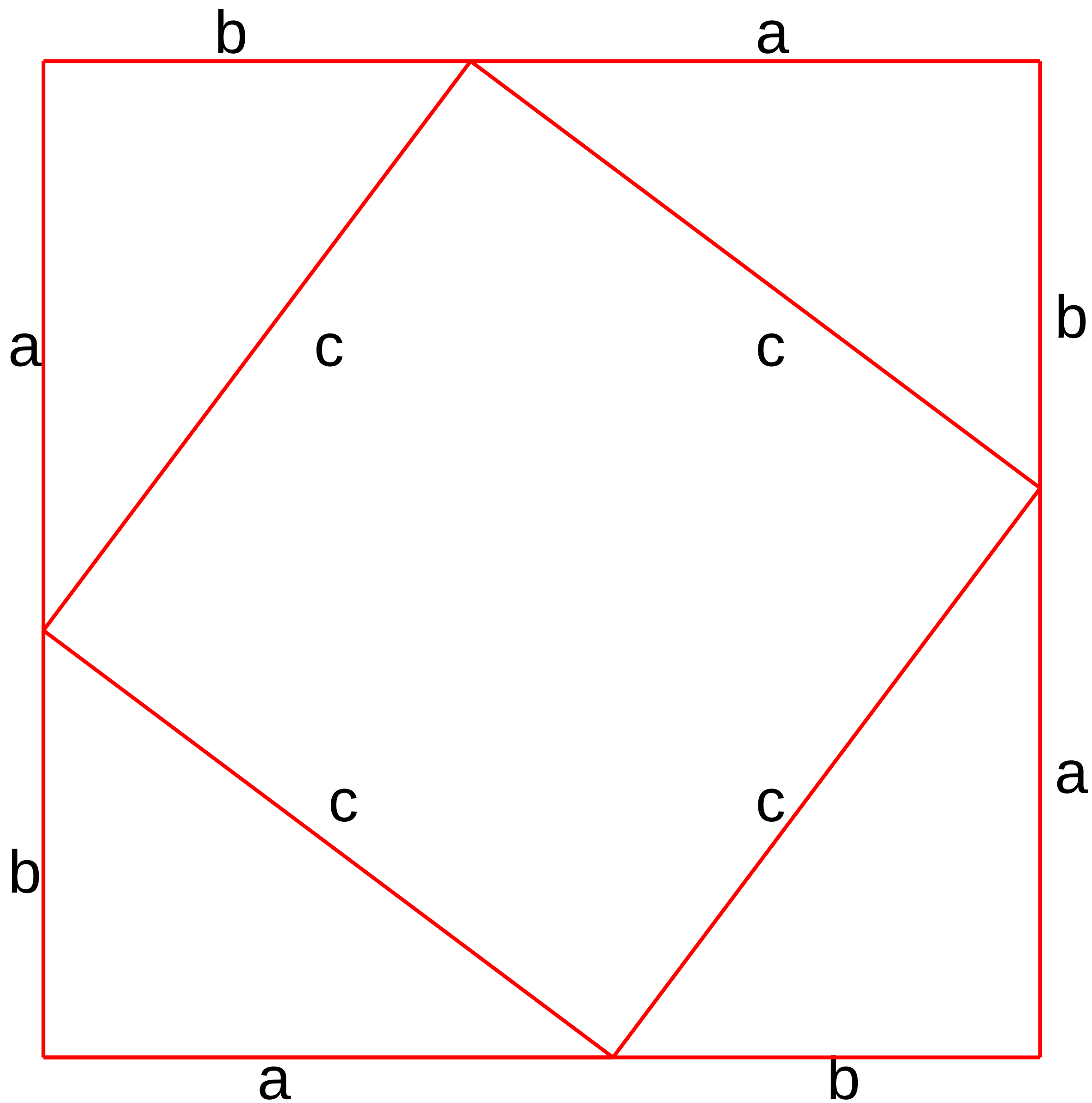


Pythagorean Theorem



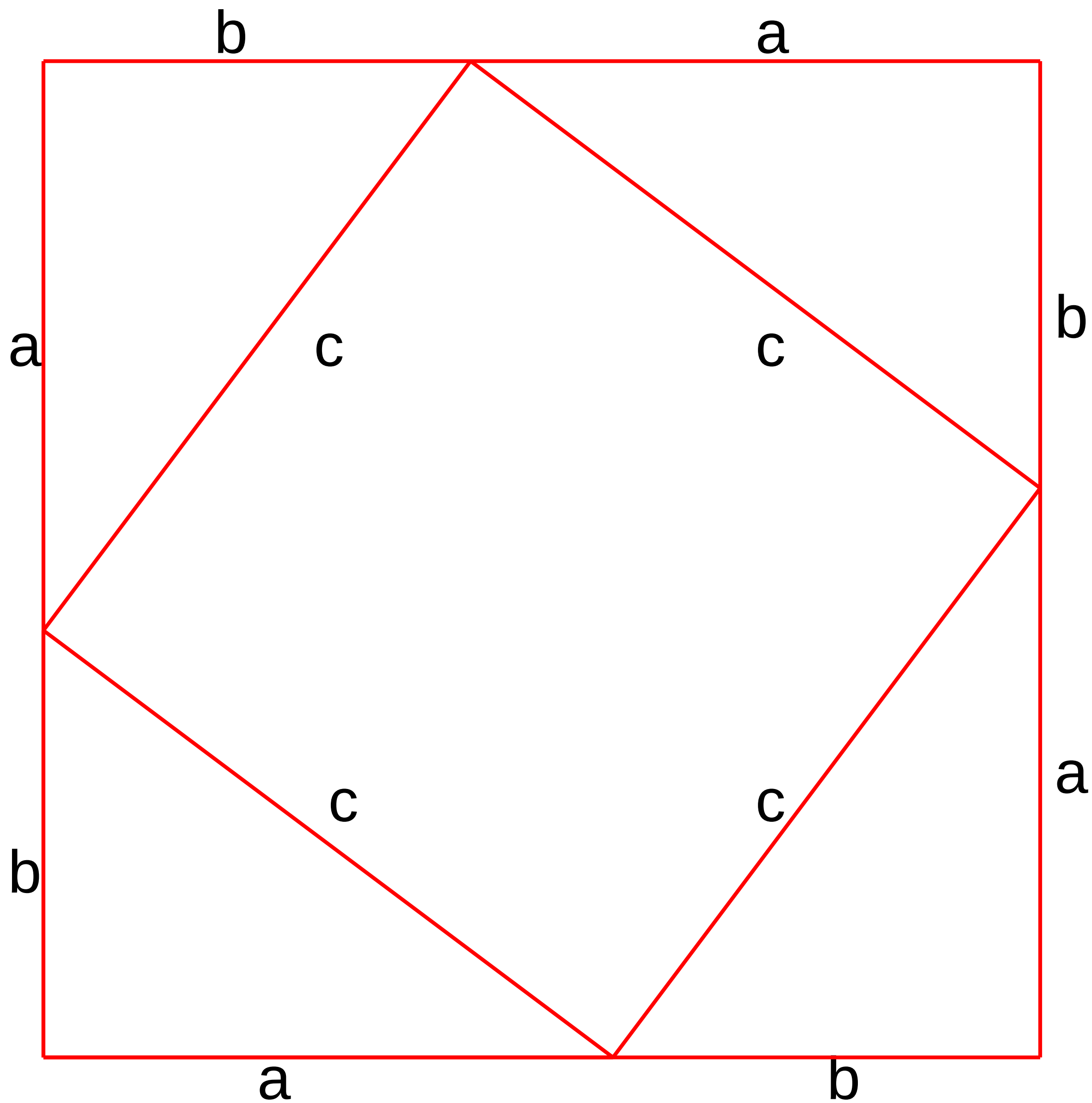
Side of the square =?

Pythagorean Theorem



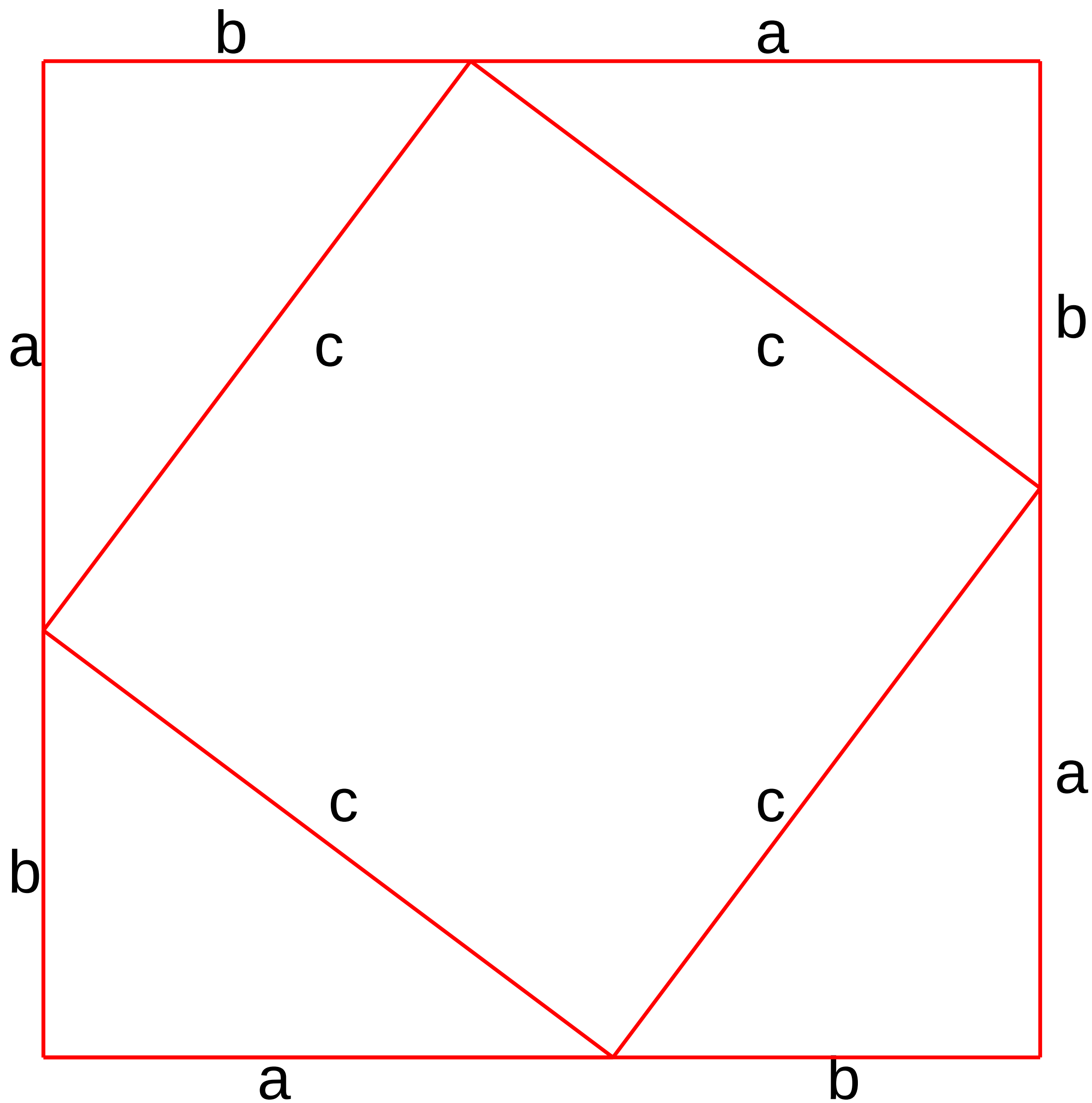
Side of the square = $(a + b)$

Pythagorean Theorem



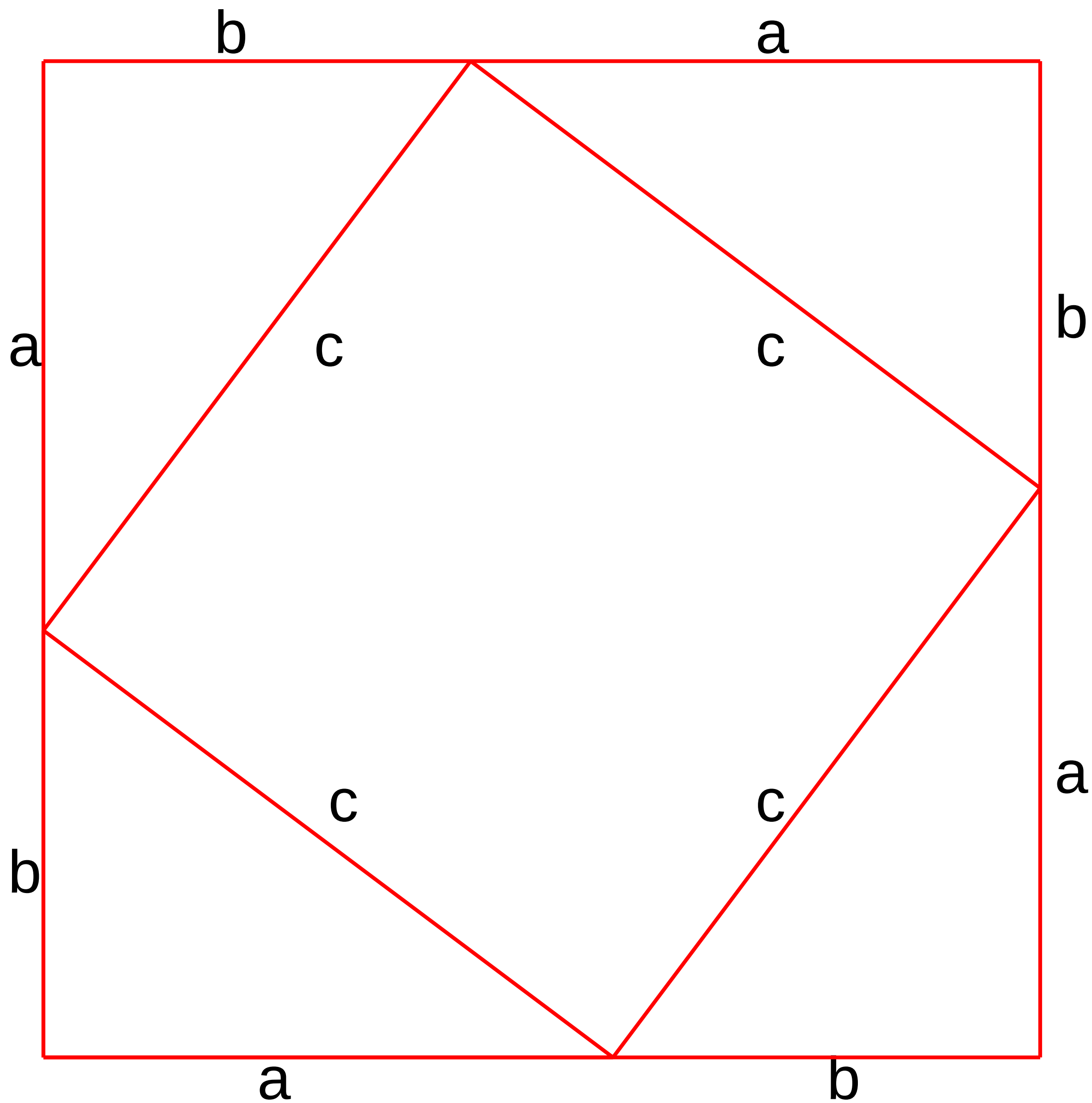
Area of the square =?

Pythagorean Theorem



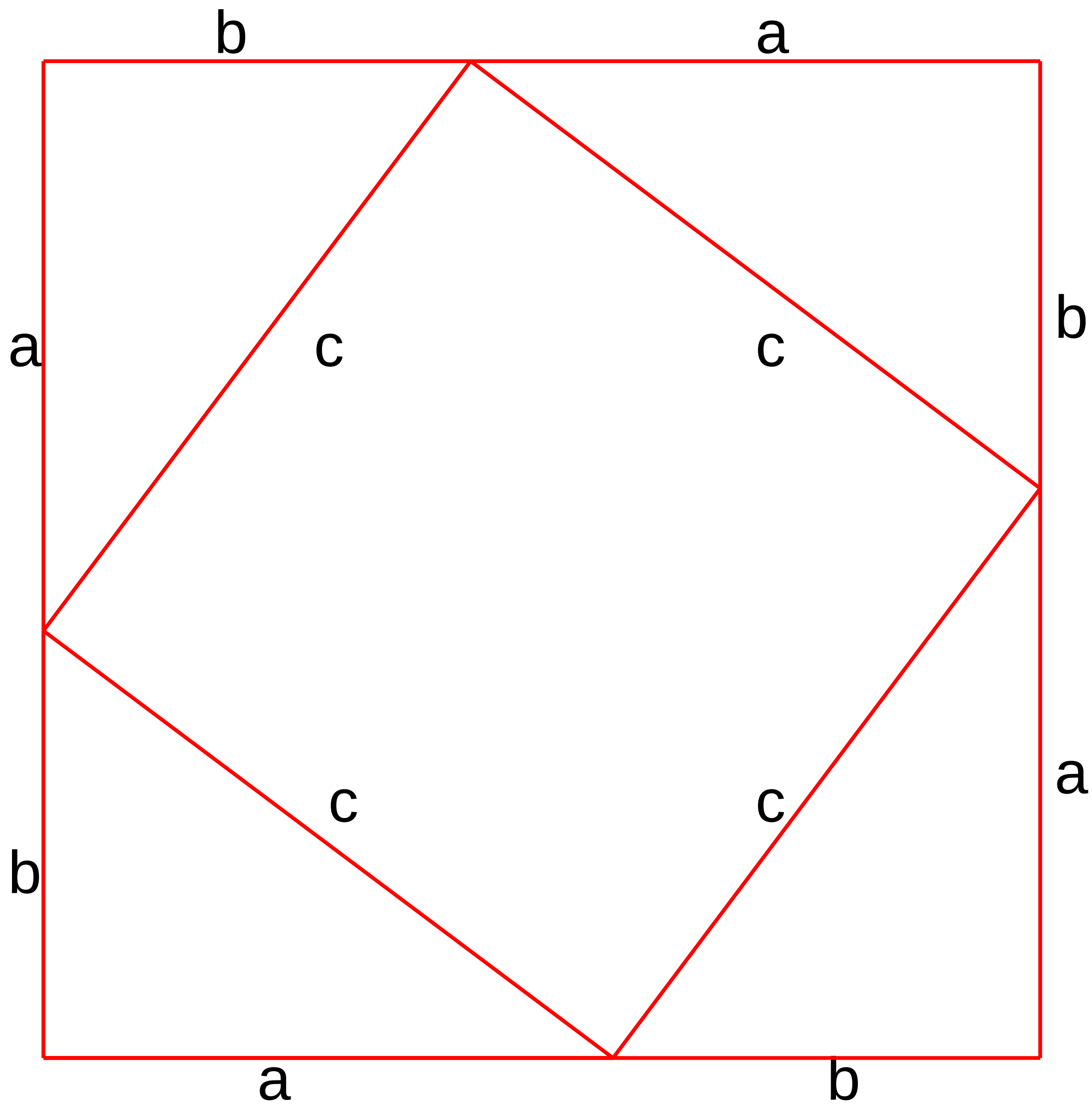
Area of the square $= (a + b)^2$

Pythagorean Theorem



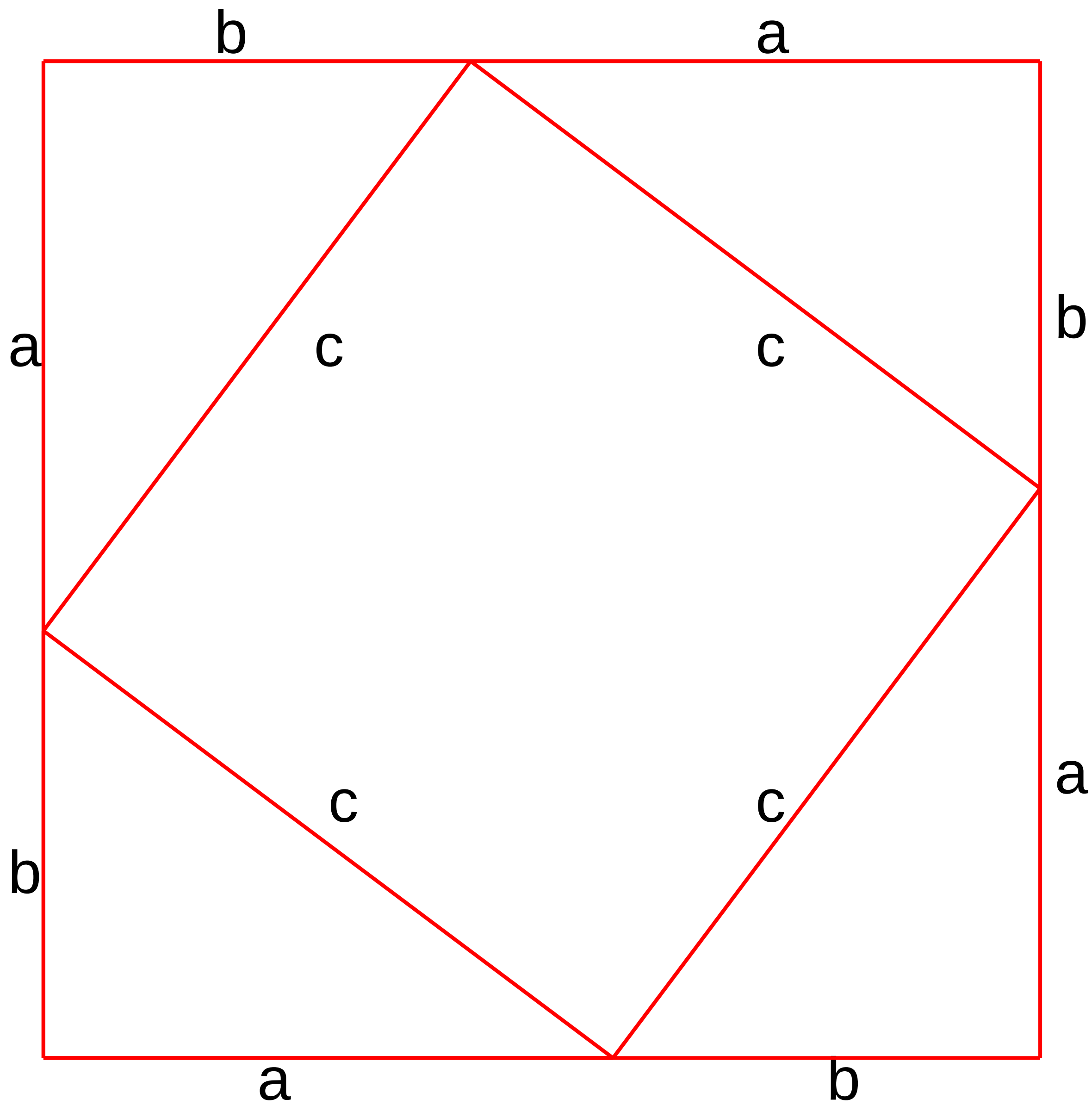
On the other hand, area consists of 4 triangles + small square

Pythagorean Theorem



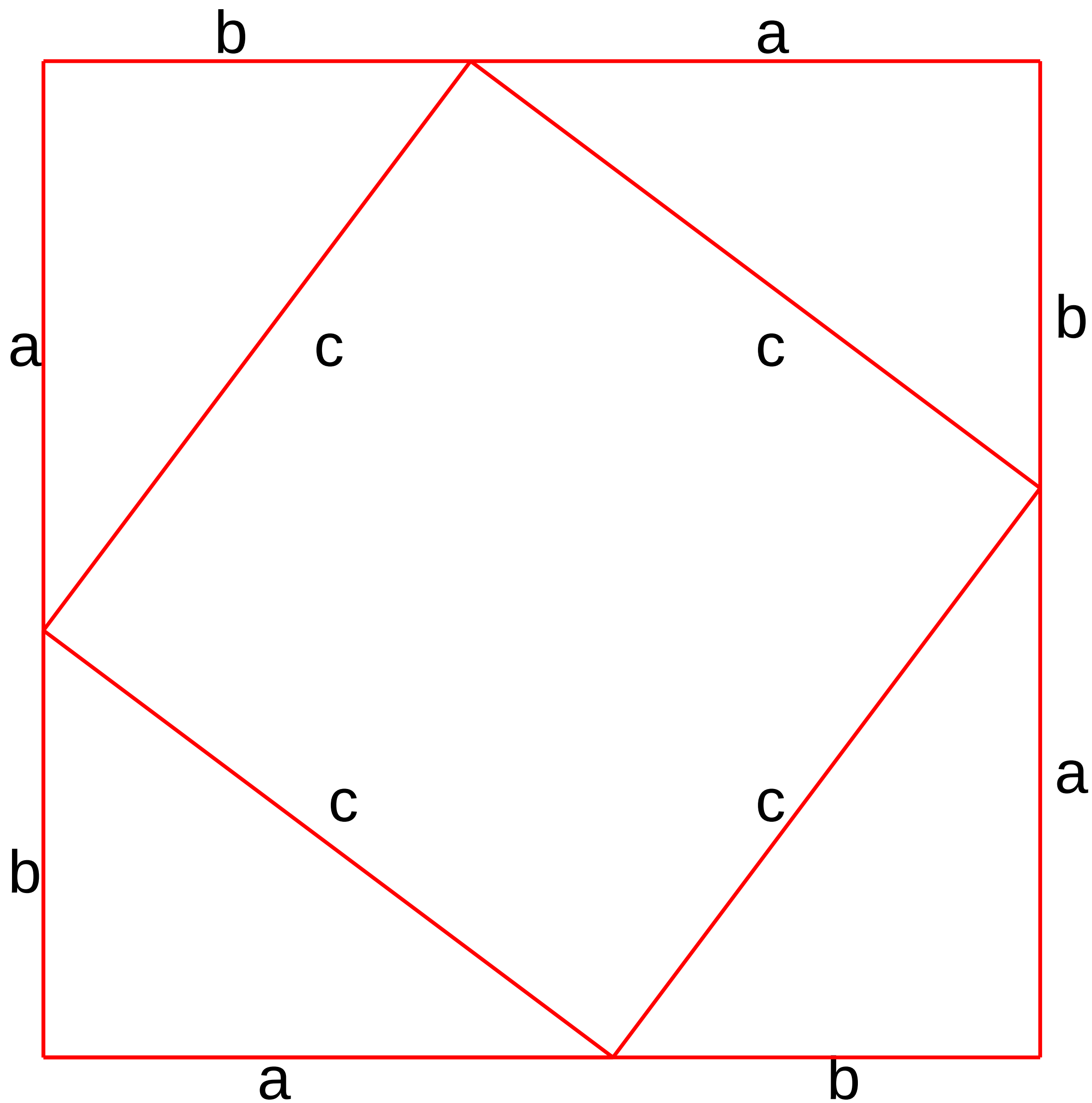
On the other hand, area consists of 4 triangles + small square.
Area of a right triangle with sides a, b, c = ?

Pythagorean Theorem



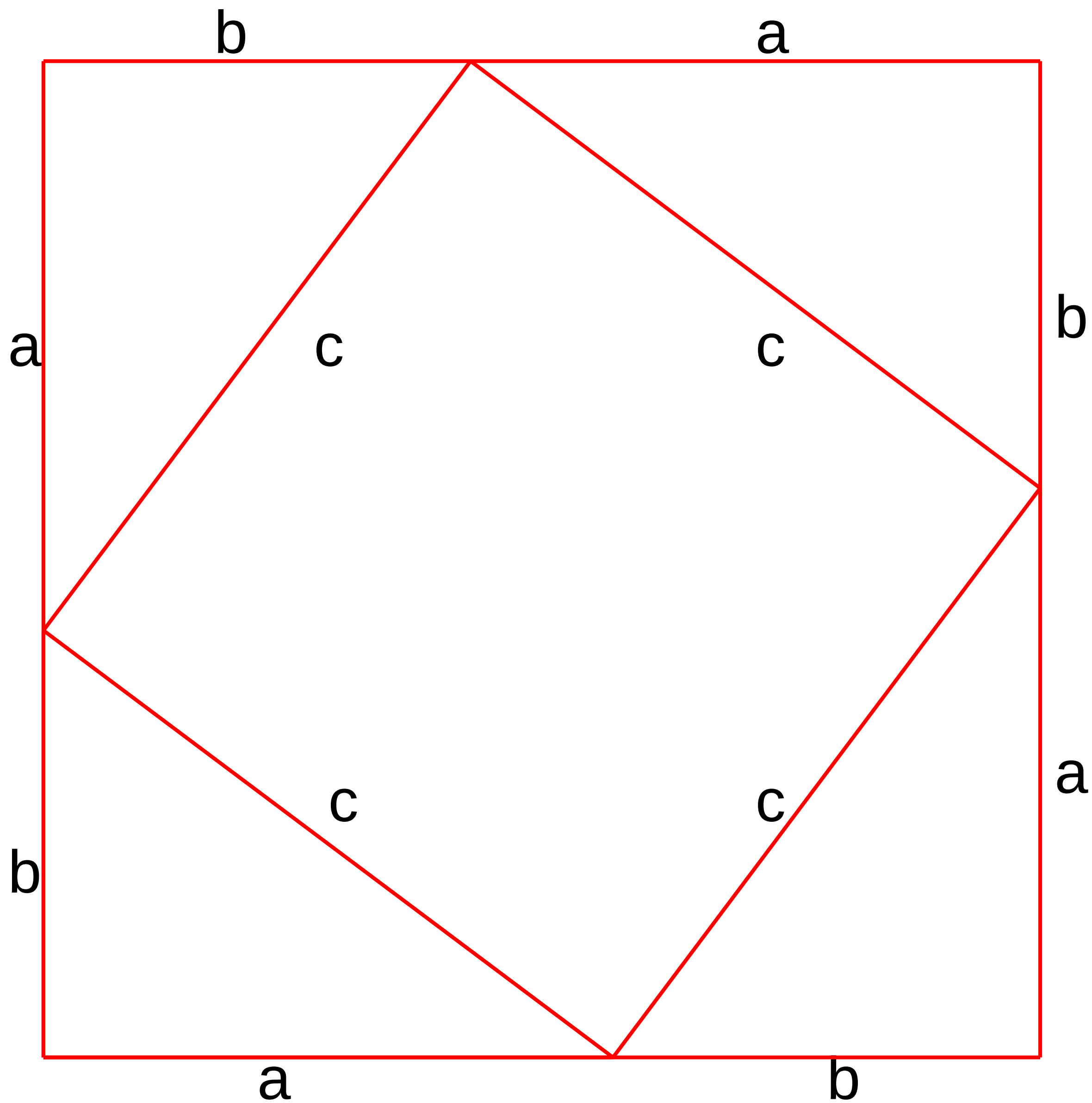
On the other hand, area consists of 4 triangles + small square.
Area of a right triangle with sides a, b, c = ab/2

Pythagorean Theorem



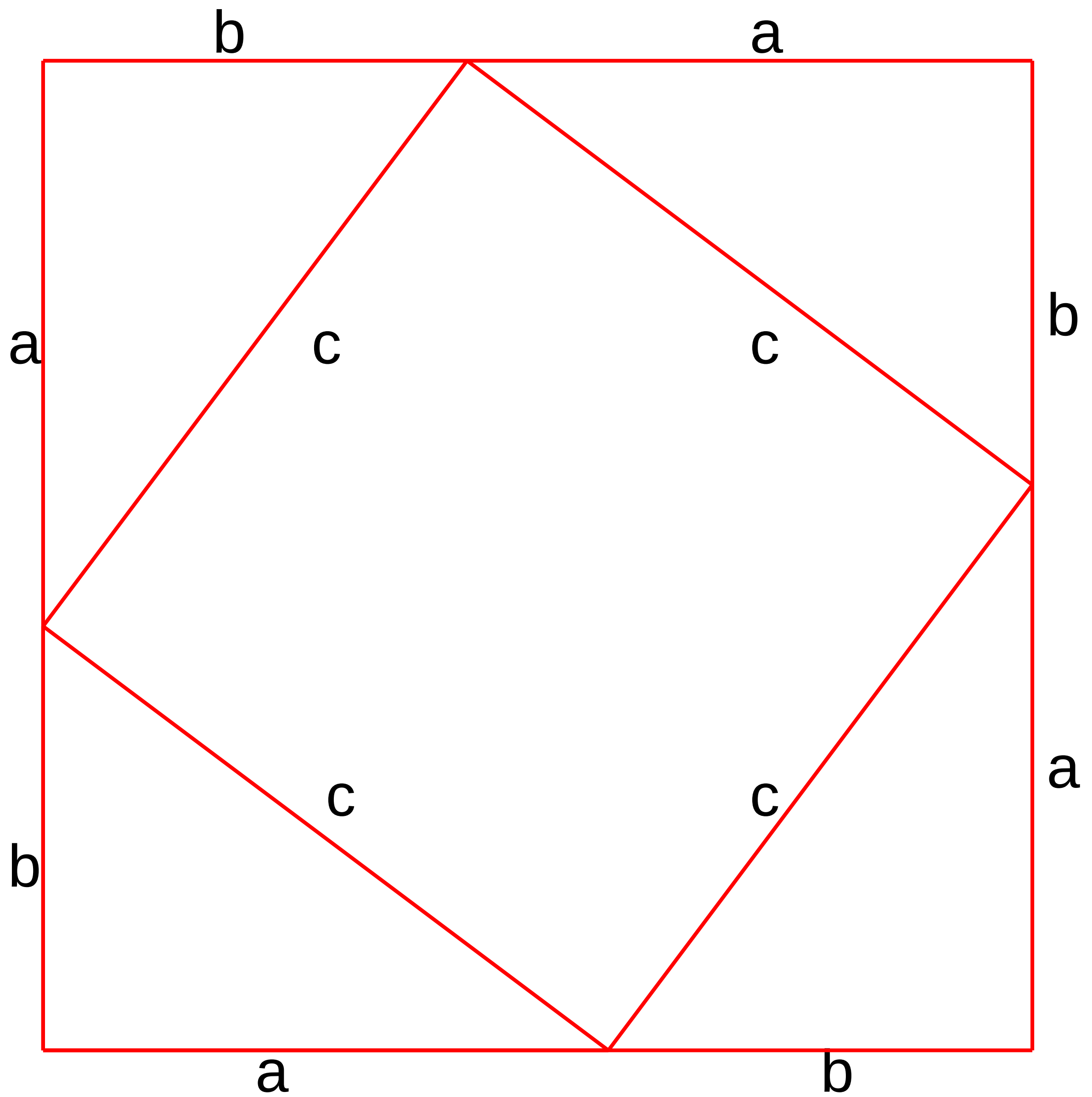
On the other hand, area consists of 4 triangles + small square.
Area of a right triangle with sides a, b, c = ab/2
Area of a square with the side c = ?

Pythagorean Theorem



On the other hand, area consists of 4 triangles + small square.
Area of a right triangle with sides a, b, c = ab/2
Area of a square with the side c = c^2

Pythagorean Theorem

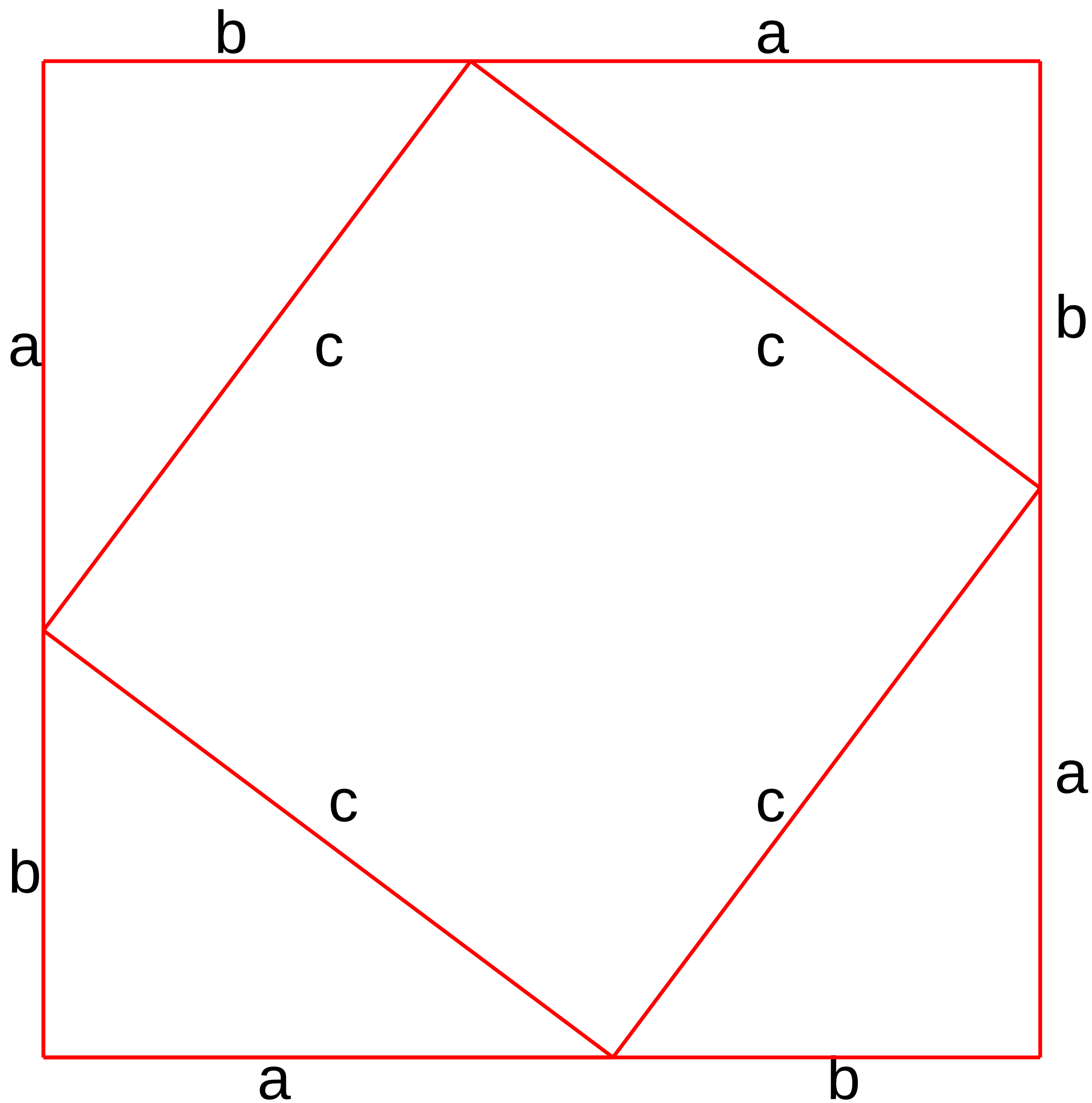


Area of a right triangle with sides a, b, c = ab/2

Area of a square with the side c = c^2

Area of the large square =

Pythagorean Theorem

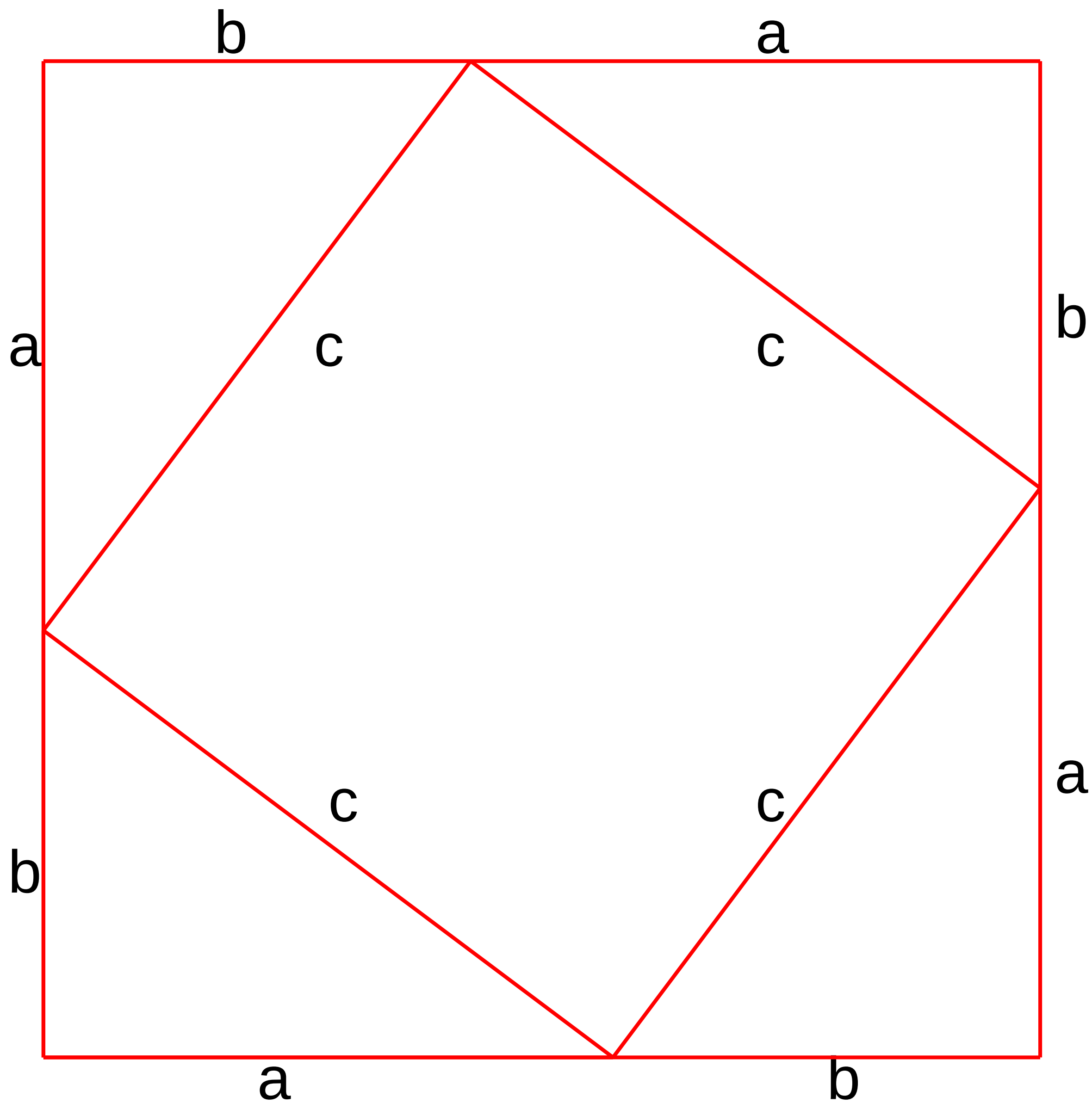


Area of a right triangle with sides a, b, c = ab/2

Area of a square with the side c = c^2

Area of the large square = 4 (ab/2) + c^2

Pythagorean Theorem

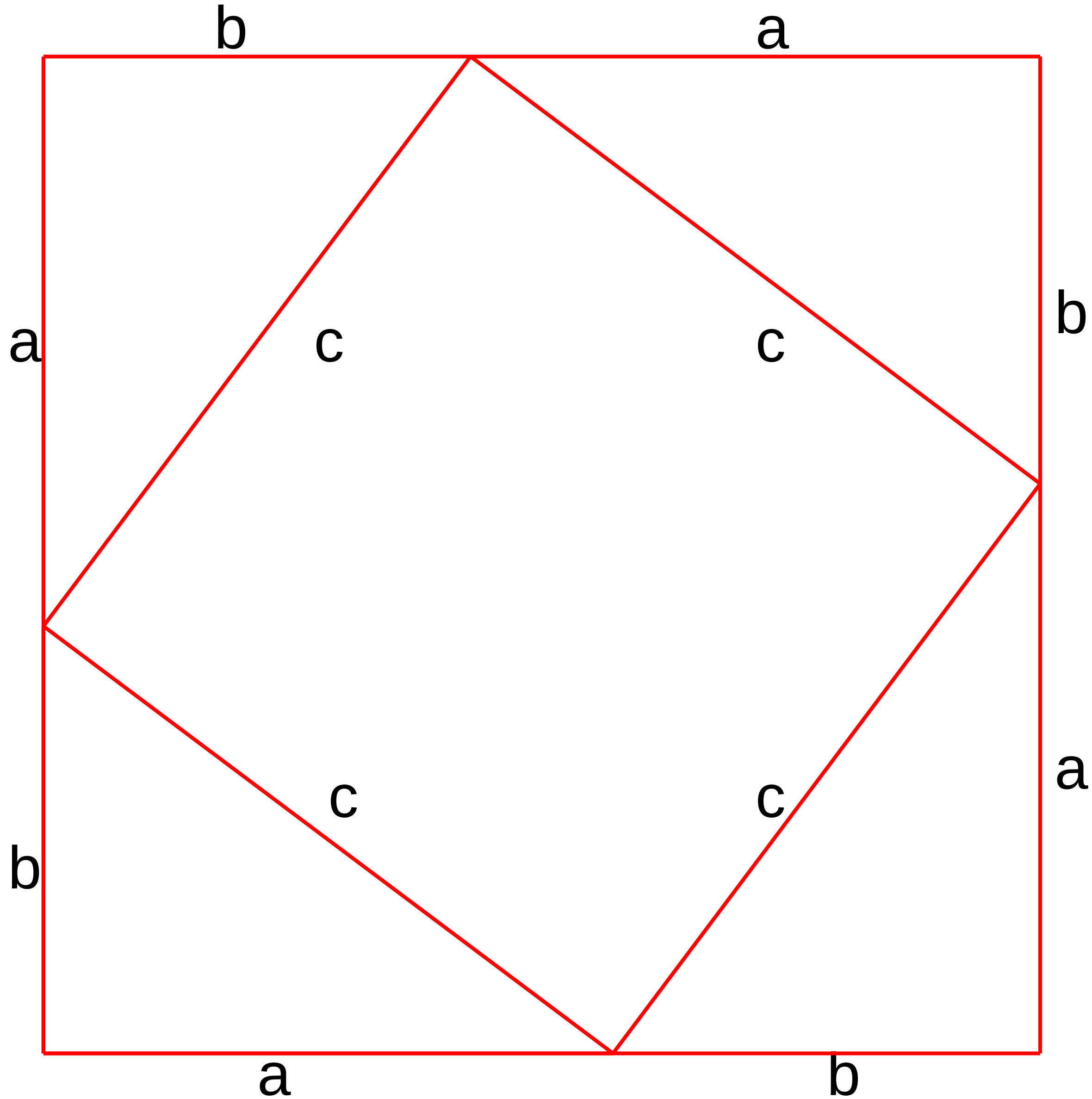


Area of a right triangle with sides a, b, c = ab/2

Area of a square with the side c = c^2

Area of the large square = 2 ab + c^2

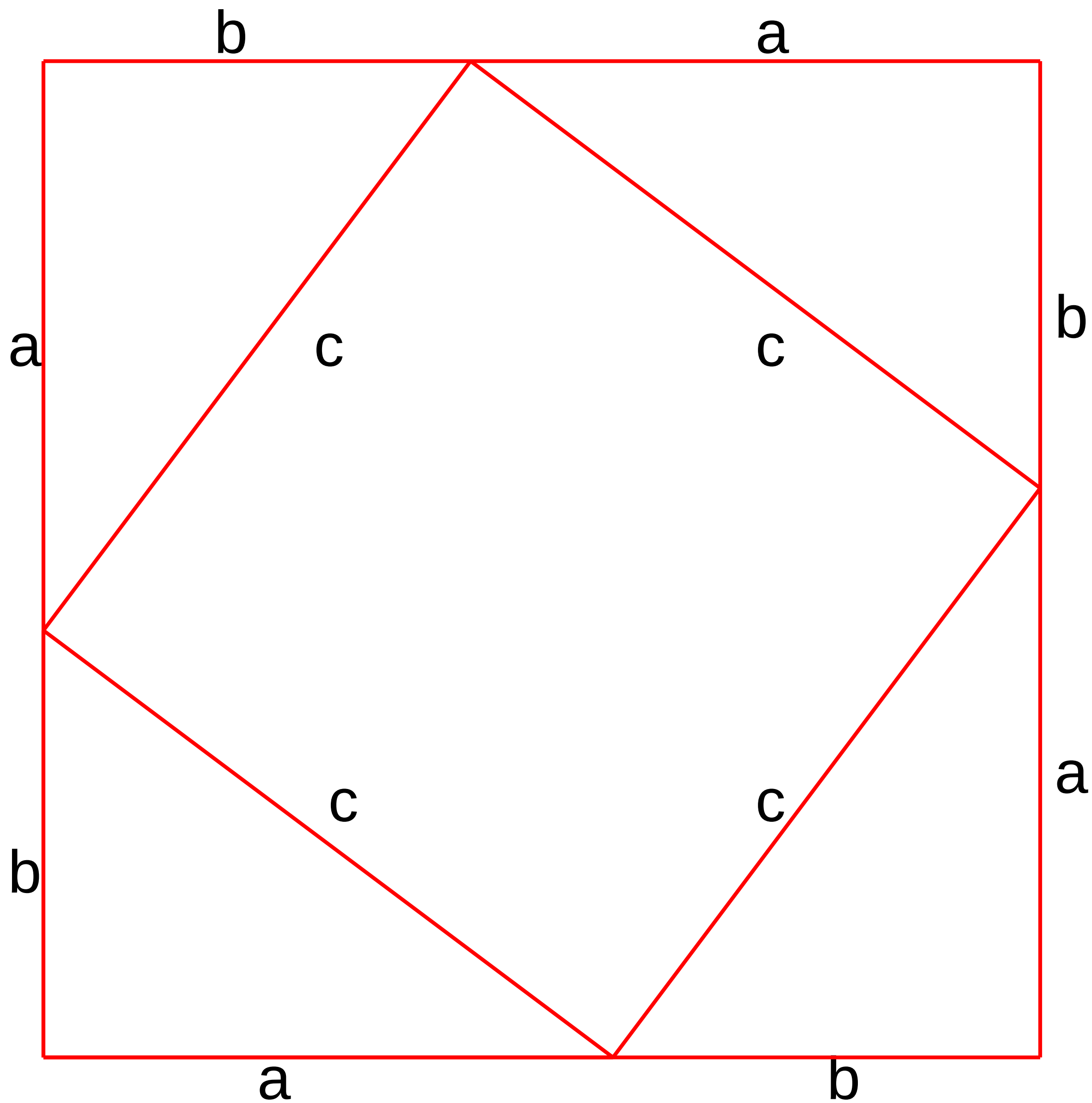
Pythagorean Theorem



Area of the large square = $2\ ab + c^2$

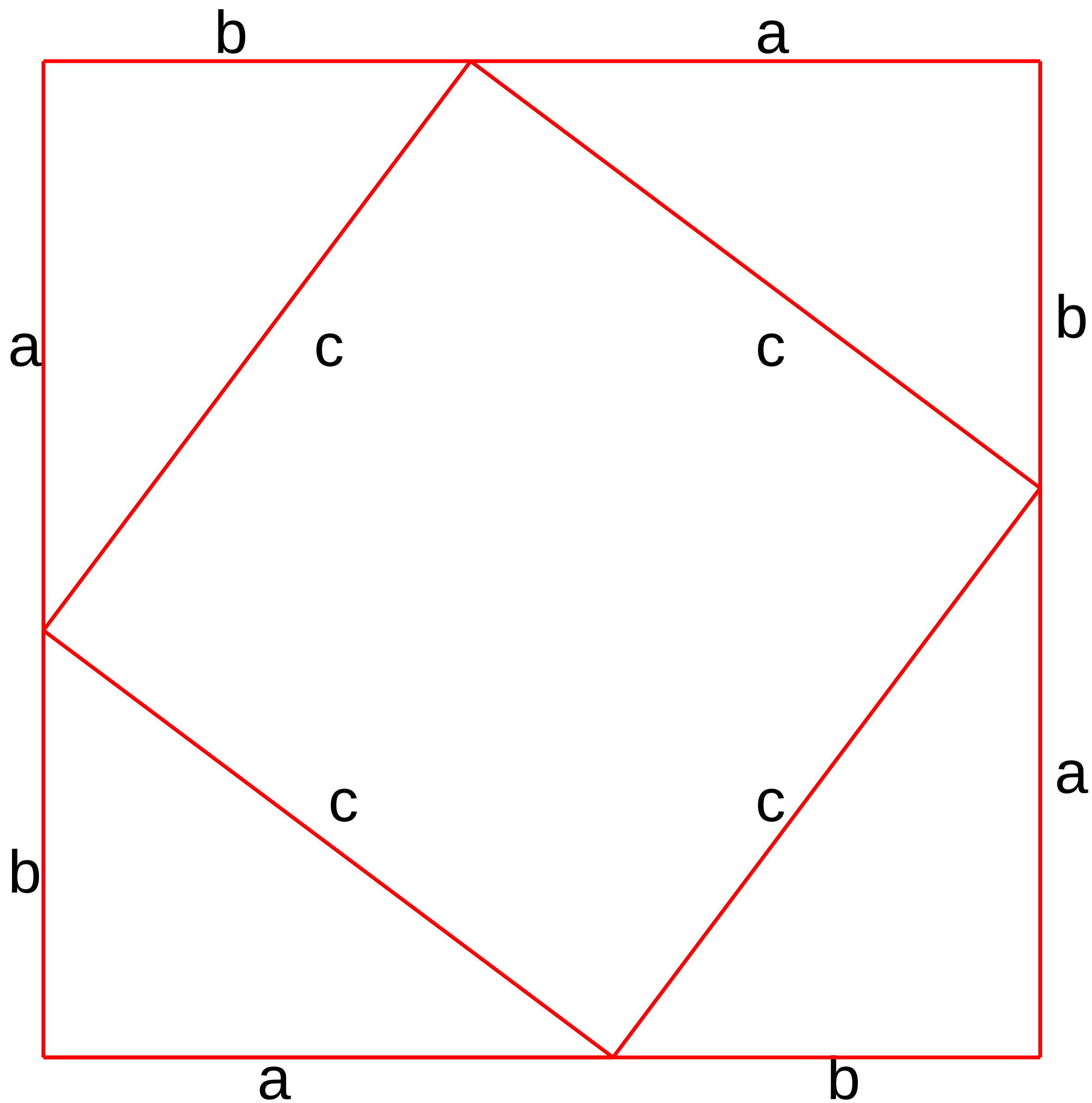
Area of the large square = $(a + b)^2$

Pythagorean Theorem



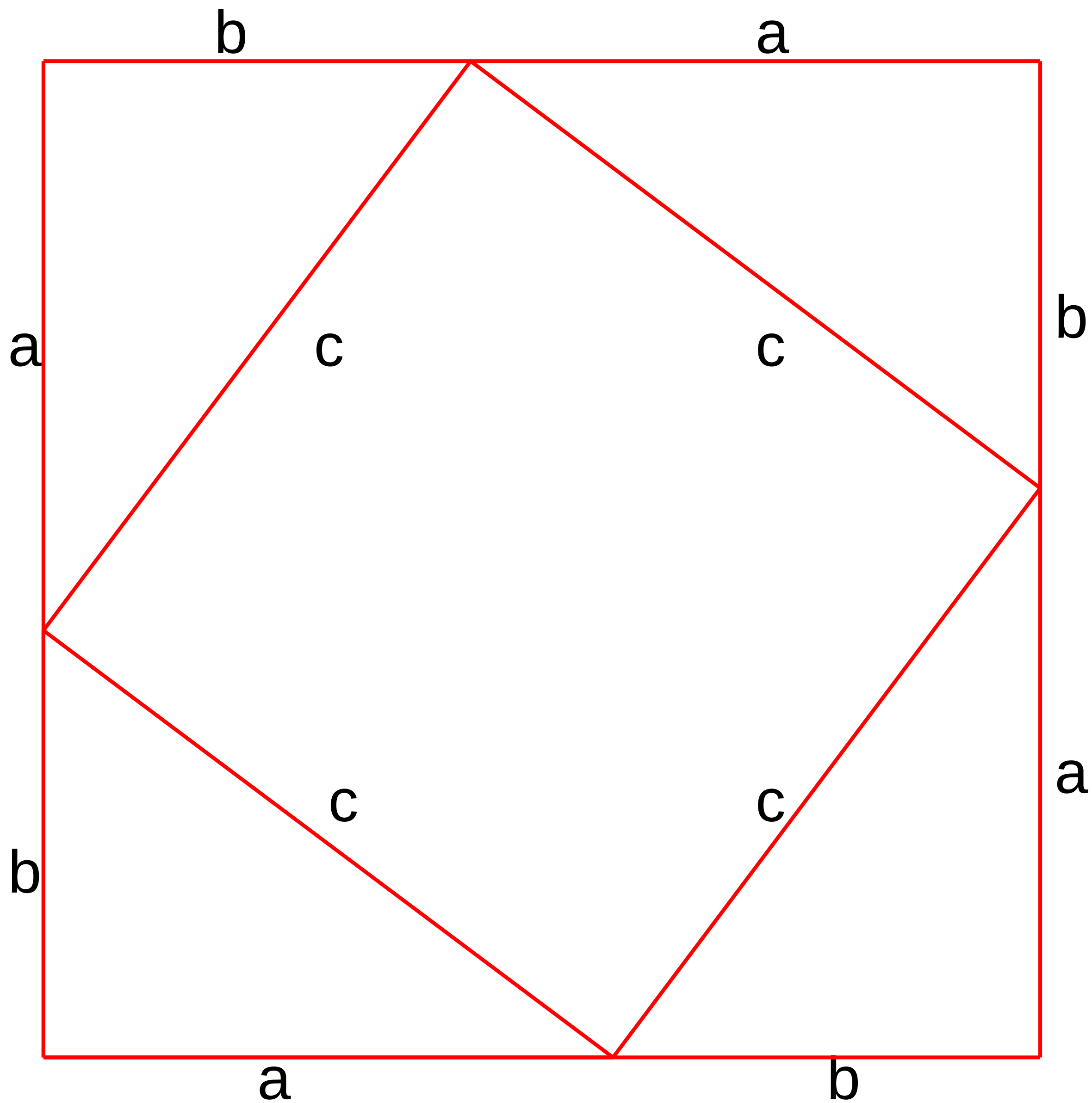
$2\ ab + c^2 = (a + b)^2$

Pythagorean Theorem



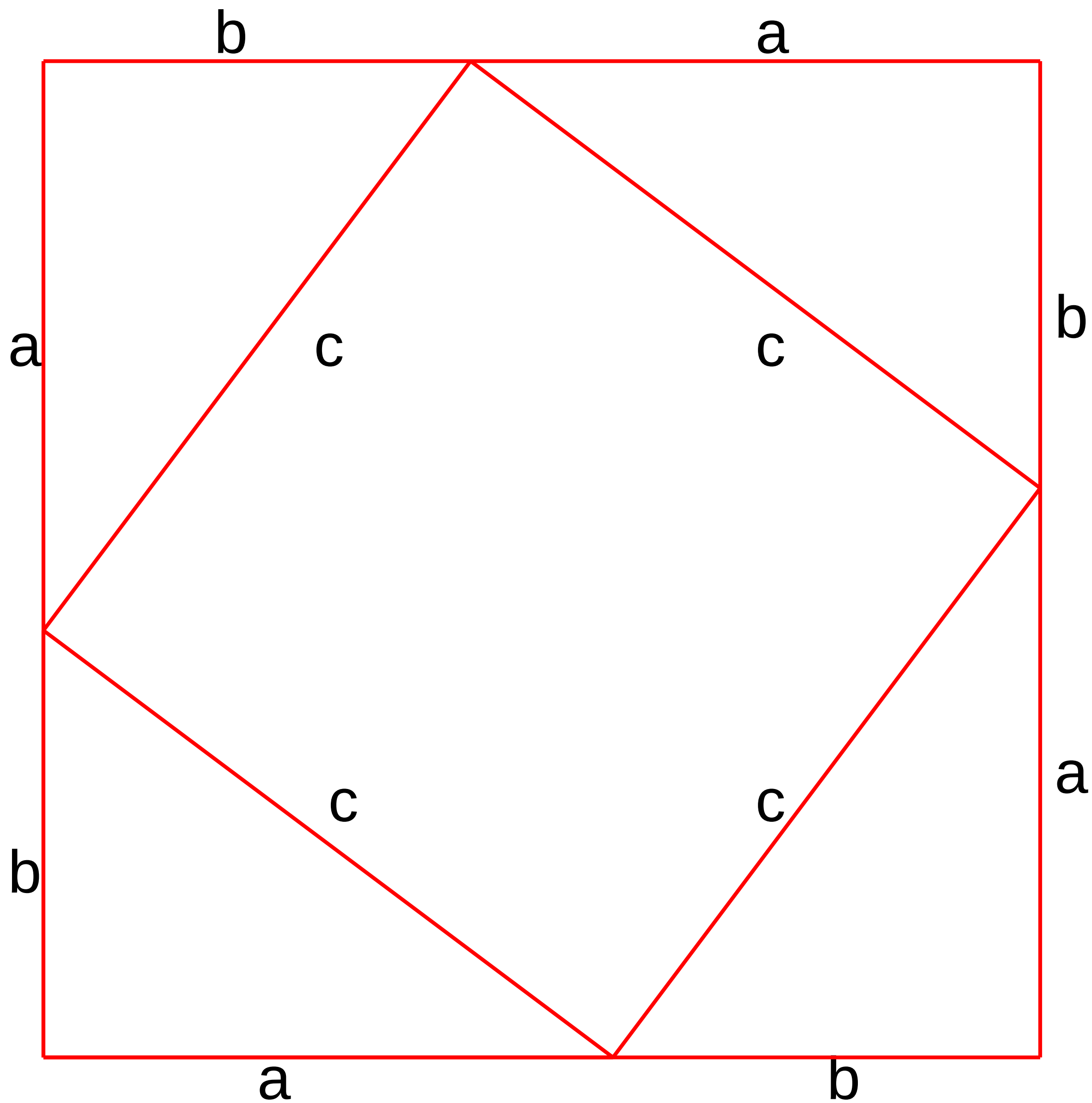
$$2\ ab + c^2 = a^2 + 2\ ab + b^2$$

Pythagorean Theorem



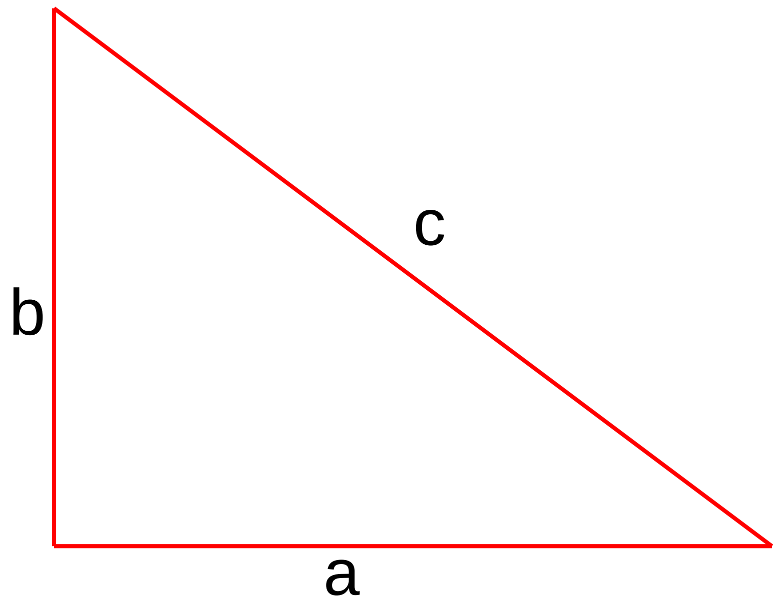
$c^2 = a^2 + b^2$

Pythagorean Theorem



$$c^2 = a^2 + b^2$$

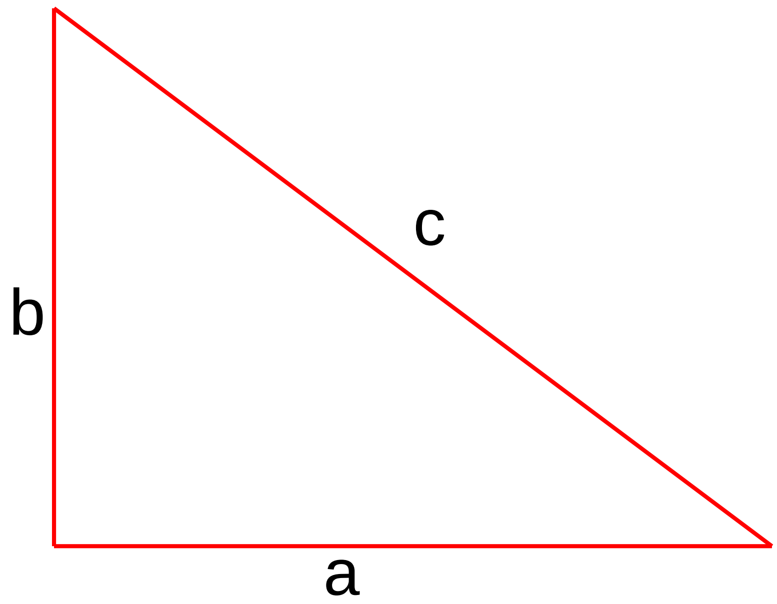
Pythagorean Theorem



a = 3, b = 4

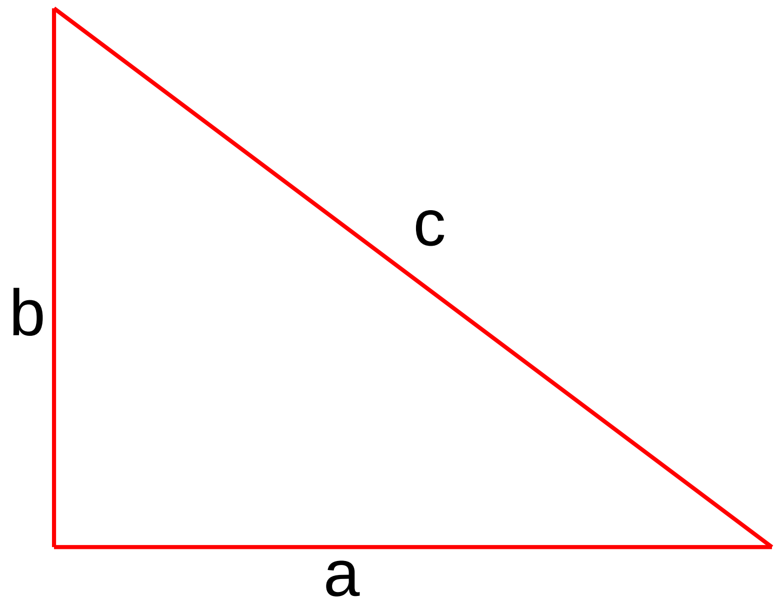
c = ?

Pythagorean Theorem



$a = 3,\ b = 4$
$c = \sqrt{3^2 + 4^2} = 5$

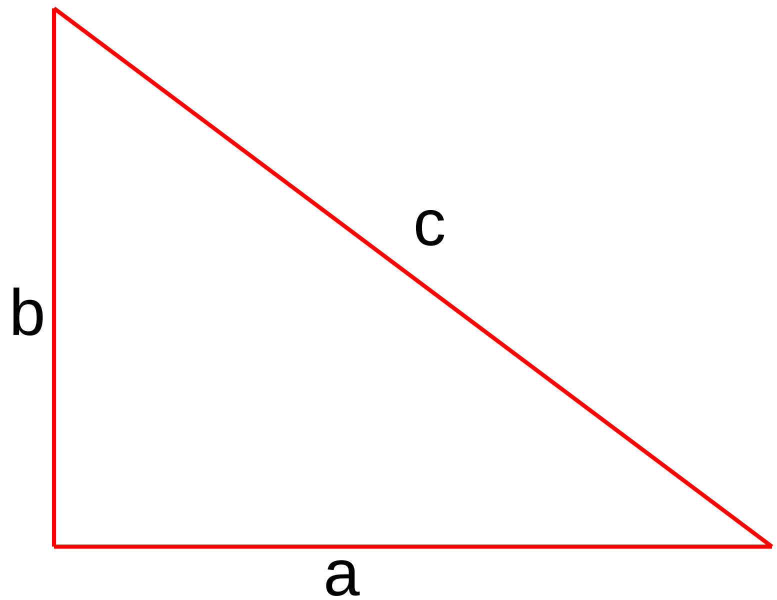
Pythagorean Theorem



$a = 5$, $b = 12$

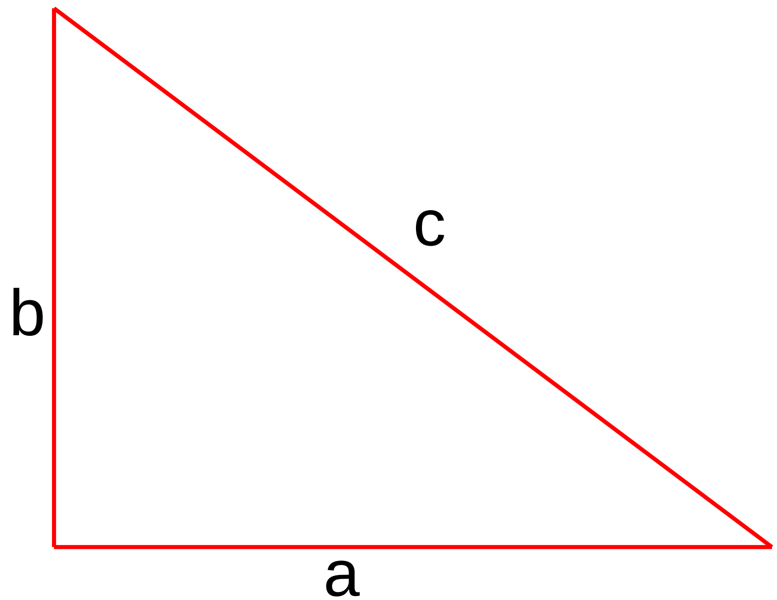
$c = ?$

Pythagorean Theorem



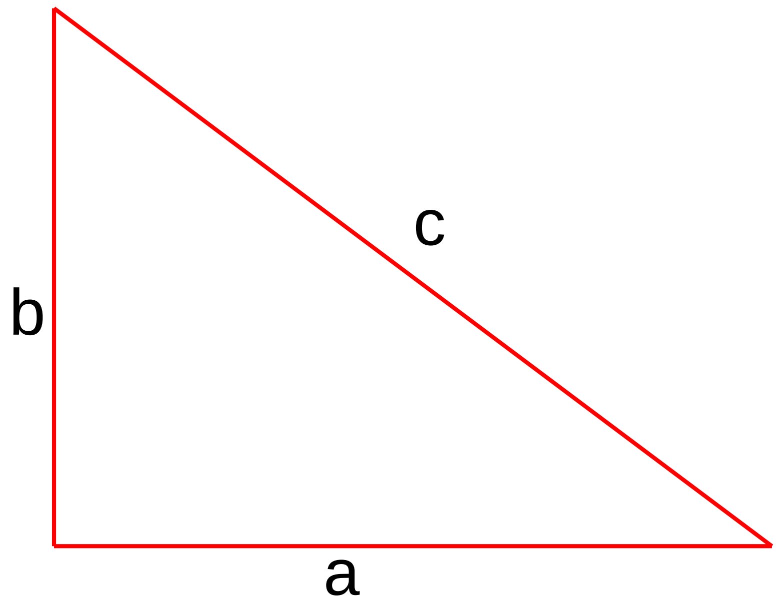
$a = 5,\ b = 12$
$c = \sqrt{5^2 + 12^2} = 13$

Pythagorean Theorem



$a = 1$, $b = 1$
$c = ?$

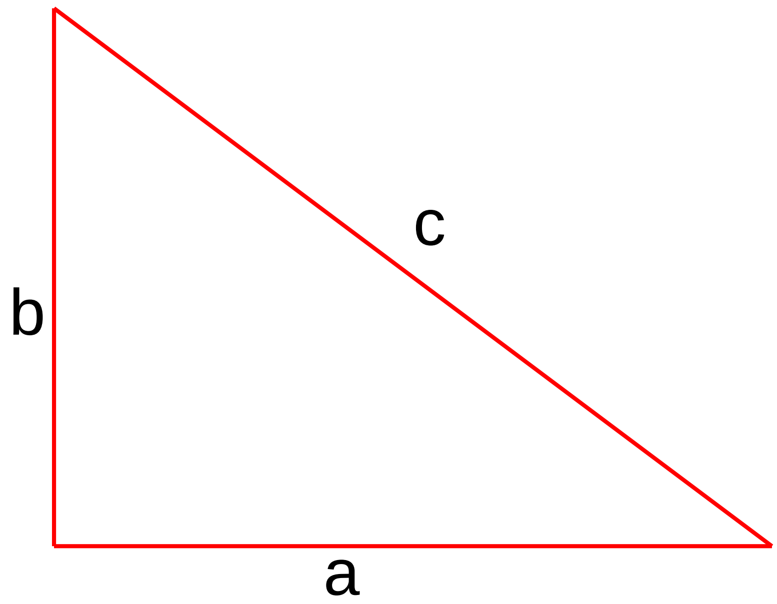
Pythagorean Theorem



$a = 1$, $b = 1$

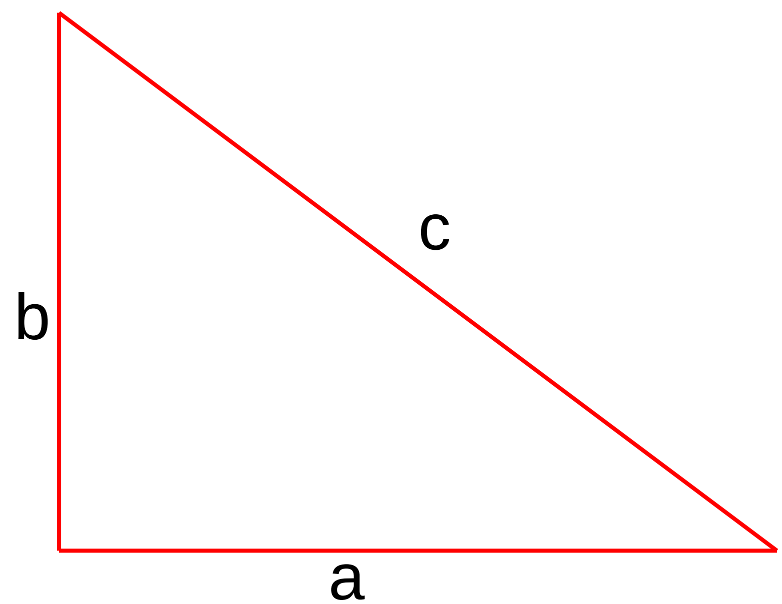
$c = \sqrt{1^2 + 1^2} = \sqrt{2}$

Pythagorean Theorem



$a = 3,\ b = 4,\ c = 5$
$a = 1,\ b = 1,\ c = \sqrt{2}$

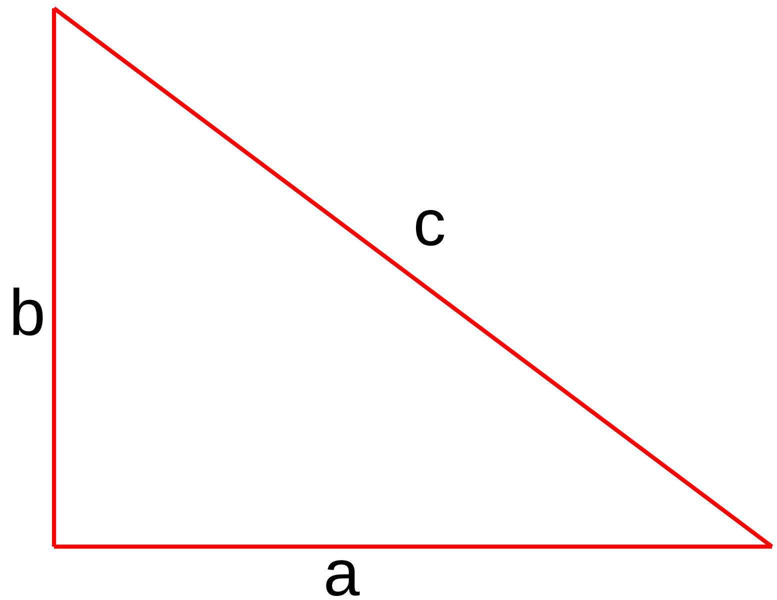
Pythagorean Theorem



$a = 3, b = 4, c = 5$
$a = 1, b = 1, c = \sqrt{2}$

When a, b, c are all integers ?

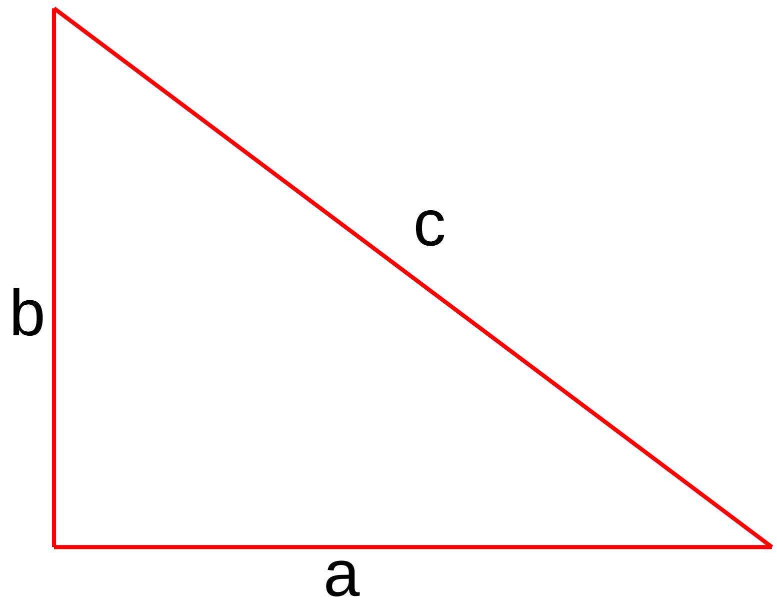
Pythagorean Triples



$c^2 = a^2 + b^2$

Pythagorean triples

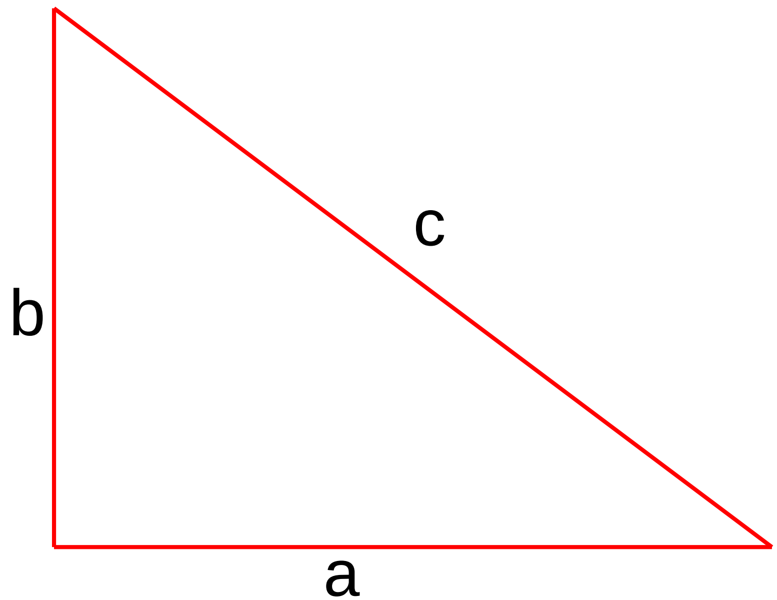
Pythagorean Triples



$c^2 = a^2 + b^2$

Euclid's formula: $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$

Pythagorean Triples

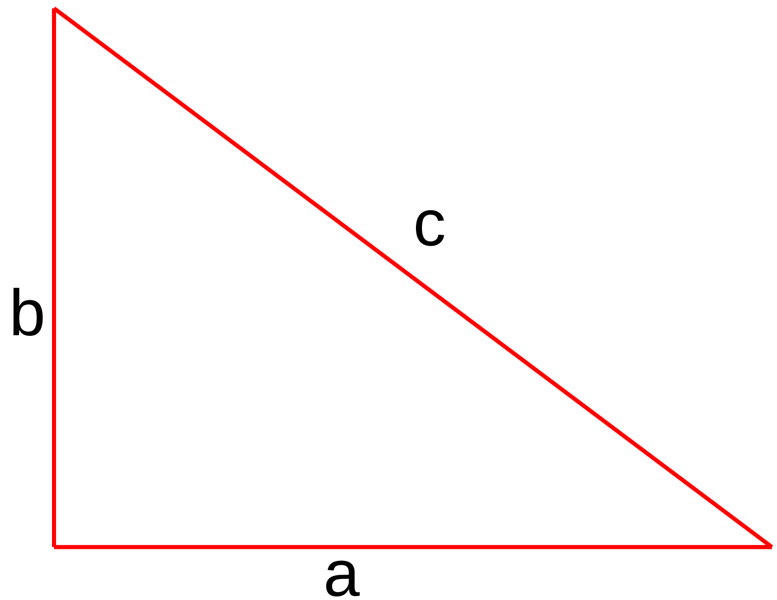


$c^2 = a^2 + b^2$

Euclid's formula: $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$

$m = 2$, $n = 1$. $a, b, c = ?$

Pythagorean Triples

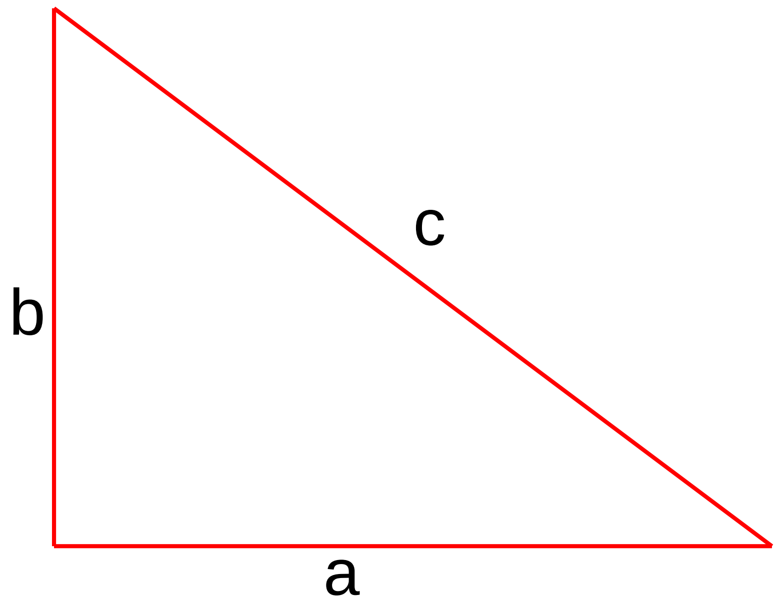


$c^2 = a^2 + b^2$

Euclid's formula: $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$

$m = 2$, $n = 1$. $a = 3$, $b = 4$, $c = 5$

Pythagorean Triples

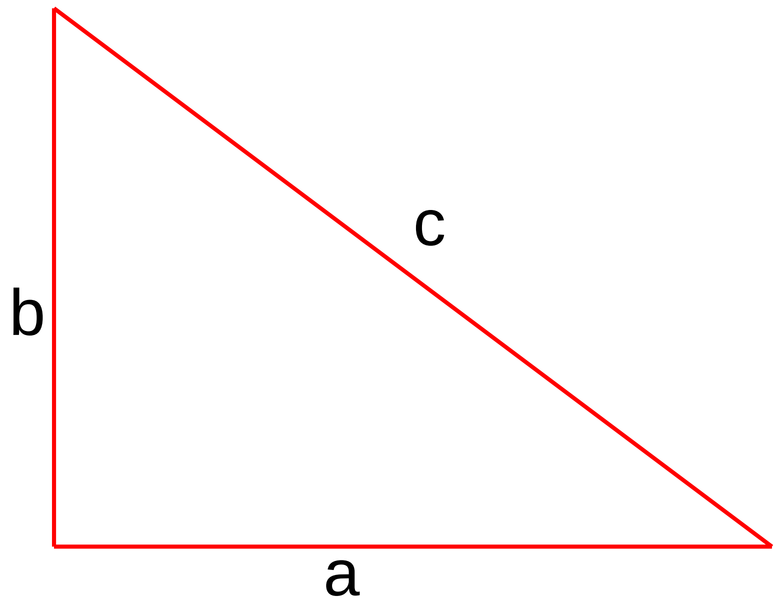


$c^2 = a^2 + b^2$

Euclid's formula: $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$

$m = 3$, $n = 1$. $a, b, c = ?$

Pythagorean Triples



$c^2 = a^2 + b^2$

Euclid's formula: $a = m^2 - n^2$, $b = 2mn$, $c = m^2 + n^2$

$m = 3$, $n = 1$. $a = 8$, $b = 6$, $c = 10$

History

From Wikipedia:

The proof is attributed to Pythagoras of Samos, c. 570 c. 495 BC - an Ionian Greek philosopher and mathematician. He is often revered as a great mathematician and scientist and is best known for the Pythagorean theorem which bears his name.

According to historians of mathematics, Pythagorean theorem was well-known to the mathematicians of the First Babylonian Dynasty (20th to 16th centuries BC), which would have been over a thousand years before Pythagoras was born.

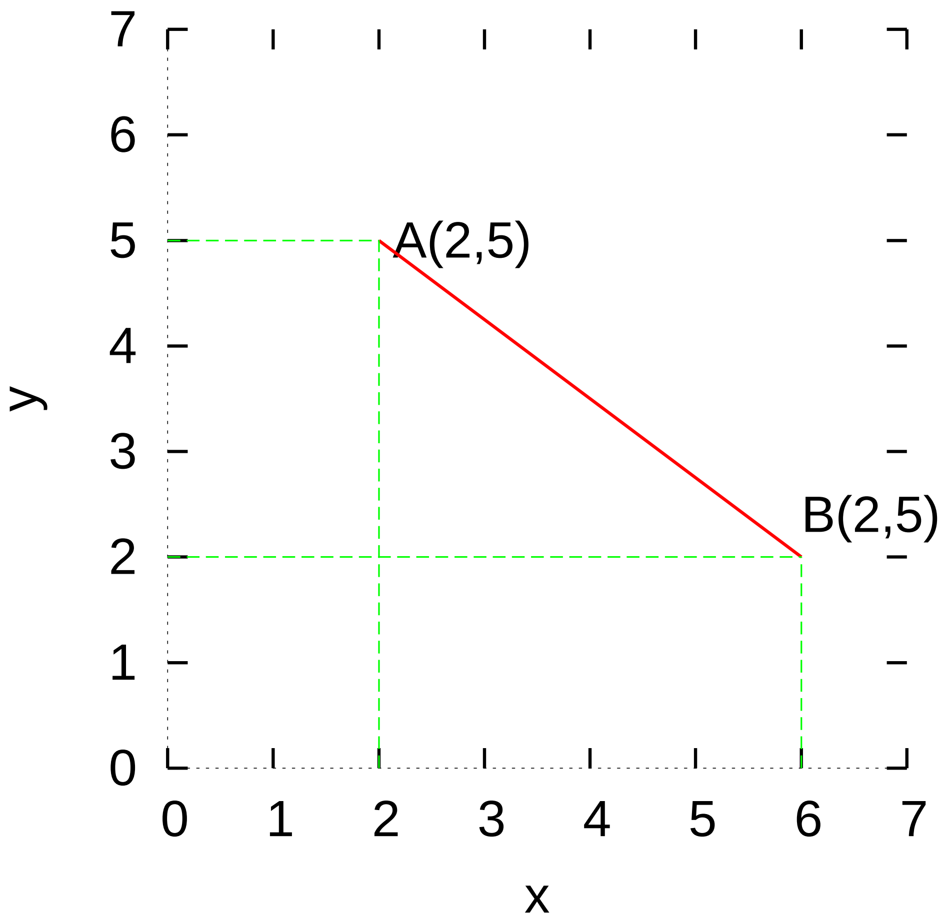
Application

Pythagorean theorem is very important in mathematics, and all other hard sciences, and technology.

Possibly the most important application is that it gives a simple formula for distance between two points in Cartesian coordinates.

If point A has coordinates (x_A, y_A), and point B has coordinates (x_B, y_B), then the distance between them is

$$\mathrm{AB} = \sqrt{(\mathrm{x_A} - \mathrm{x_B})^2 + (\mathrm{y_A} - \mathrm{y_B})^2}$$



Application

Similar formula holds in 3 dimensions:
If point A has coordinates (x_A, y_A, z_A), and point B has coordinates (x_B, y_B, z_B), then the distance between them is

$$\mathrm{AB} = \sqrt{(\mathrm{x_A} - \mathrm{x_B})^2 + (\mathrm{y_A} - \mathrm{y_B})^2 + (\mathrm{z_A} - \mathrm{z_B})^2}$$

Similarly for complex numbers, the absolute value of $\mathrm{z} = \mathrm{x} + \mathrm{iy}$ is

$$|\mathrm{z}| = \sqrt{\mathrm{x}^2 + \mathrm{y}^2},$$

which is the distance from origin (0, 0) to the point (x, y).

QUIZ

In right triangle with two of the sides given, calculate the remaining side.

a = 3, b = 4. c =
a = 6, b = 8. c =
a = 5, c = 13. b =
a = 15, c = 17. b =

Construct Pythagorean triple $a = m^2 - n^2$, $b = 2\,mn$, $c = m^2 + n^2$:
m = 2, n = 1. a = , b = , c =
m = 3, n = 2. a = , b = , c =

Calculate distance between point A(x=2,y=5), and B(x=6,y=2)