UCDAVIS

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Adding Fractions May 17, 2017

Adding Fractions

Outline

- Last Time
- Review Quiz
- The Problem: Combining Fractions
- Solution: Least Common Denominator
- Exercises
- Inverse Problem: "Decombining" Fractions
- QUIZ

Adding Fractions

$$\frac{2}{7} + \frac{3}{7} =$$

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$$\frac{2}{7} + \frac{3}{7} = \frac{2+3}{7} = \frac{5}{7}$$

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$$\frac{x}{(x+1)(x+2)} + \frac{5}{(x+1)(x+2)} =$$

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Adding Fractions

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$$\frac{x}{(x+1)} + \frac{5}{(x+2)} =$$

- Cannot add right away - different denominators

Recall: Equivalent Fractions

$$\frac{a}{b}$$

We can multiply numerator and denominator by any expression (except 0), and fraction will remain the same

Recall: Equivalent Fractions

$$\frac{a}{b} = \frac{ac}{bc}$$

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$$\frac{a}{b} = \frac{ac}{bc}$$

$$\frac{x}{x+1} = \frac{x(x+2)}{(x+1)(x+2)}$$

Common Denominator

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First fraction

$$\frac{x}{(x+1)} =$$

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$$\frac{x}{(x+1)} = \frac{x(x+2)}{(x+1)(x+2)}$$

Common Denominator

$$\frac{x}{(x+1)} + \frac{5}{(x+2)} =$$

We want both fractions have the same denominator
First fraction

$$\frac{x}{(x+1)} = \frac{x(x+2)}{(x+1)(x+2)}$$

Second fraction

$$\frac{5}{(x+2)} =$$

Common Denominator

$$\frac{x}{(x+1)} + \frac{5}{(x+2)} =$$

We want both fractions have the same denominator
First fraction

$$\frac{x}{(x+1)} = \frac{x(x+2)}{(x+1)(x+2)}$$

Second fraction

$$\frac{5}{(x+2)} = \frac{5(x+1)}{(x+2)(x+1)}$$

Common Denominator

$$\frac{x}{(x+1)} + \frac{5}{(x+2)} =$$

We want both fractions have the same denominator
First fraction

$$\frac{x}{(x+1)} = \frac{x(x+2)}{(x+1)(x+2)}$$

Second fraction

$$\frac{5}{(x+2)} = \frac{5(x+1)}{(x+2)(x+1)}$$

Now we have the same denominator

$$\frac{x(x+2)}{(x+1)(x+2)} + \frac{5(x+1)}{(x+2)(x+1)} =$$

Common Denominator

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Second fraction

$$\frac{5}{(x+2)} = \frac{5(x+1)}{(x+2)(x+1)}$$

Now we have the same denominator

$$\frac{x(x+2)}{(x+1)(x+2)} + \frac{5(x+1)}{(x+2)(x+1)} = \frac{x(x+2)+5(x+1)}{(x+1)(x+2)} = \frac{x^2+2x+5x+5)}{(x+1)(x+2)}$$

$$= \frac{x^2+7x+5)}{(x+1)(x+2)}$$

Common Denominator

$$\frac{x}{y} - \frac{y}{x} =$$

Common Denominator

$$\frac{x}{y} - \frac{y}{x} =$$

$$\frac{x}{y} = \frac{x^2}{xy}$$

$$\frac{y}{x} = \frac{y^2}{xy}$$

Common Denominator

$$\frac{x}{y} - \frac{y}{x} =$$

$$\frac{x}{y} = \frac{x^2}{xy}$$

$$\frac{y}{x} = \frac{y^2}{xy}$$

$$\frac{x}{y} - \frac{y}{x} = \frac{x^2}{xy} - \frac{y^2}{xy} = \frac{x^2 - y^2}{xy}$$

Common Denominator

How to get Common Denominator?

$$\frac{a}{b} + \frac{c}{d} =$$

Common Denominator

How to get Common Denominator?

$$\frac{a}{b} + \frac{c}{d} =$$

The simplest:

$$\frac{a}{b} = \frac{ad}{bd}$$

$$\frac{c}{d} = \frac{bc}{bd}$$

Common Denominator

How to get Common Denominator?

$$\frac{a}{b} + \frac{c}{d} =$$

The simplest:
Multiply numerator and denominator of the first fraction by denominator of the second fraction
Multiply numerator and denominator of the second fraction by denominator of the first fraction

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd}$$

Least Common Denominator

We want the **Least** Common Denominator

$$\frac{x+2}{x^2(x+1)} + \frac{1}{x(x+2)} =$$

Least Common Denominator

$$\frac{x+2}{x^2(x+1)} + \frac{1}{x(x+2)} =$$

Factor first denominator

$$x^2(x+1)$$

Factor second denominator

$$x(x+2)$$

Least Common Denominator

$$\frac{x+2}{x^2(x+1)} + \frac{1}{x(x+2)} =$$

Factor first denominator

$$x^2(x+1)$$

Factor second denominator

$$x(x+2)$$

Common factor x

Least Common Denominator

$$\frac{x+2}{x^2(x+1)} + \frac{1}{x(x+2)} =$$

Factor first denominator

$$x^2(x+1)$$

Factor second denominator

$$x(x+2)$$

Common factor x
Multiply both denominators and divide by common factor:

$$LCD = x^2(x+1)x(x+2)/x = x^2(x+1)(x+2)$$

Least Common Denominator

$$\frac{x+2}{x^2(x+1)} + \frac{1}{x(x+2)} =$$

Factor first denominator

$$x^2(x+1)$$

Factor second denominator

$$x(x+2)$$

Common factor x
Multiply both denominators and divide by common factor:

$$LCD = x^2(x+1)x(x+2)/x = x^2(x+1)(x+2)$$

$$\frac{x+2}{x^2(x+1)} + \frac{1}{x(x+2)} = \frac{(x+2)(x+2)+x(x+1)}{x^2(x+1)(x+2)}$$

Least Common Denominator

$$\frac{x+2}{x(x+1)} + \frac{1}{2x(x+2)} =$$

Least Common Denominator

$$\frac{x+2}{x(x+1)} + \frac{1}{2x(x+2)} =$$

$$LCD = 2x(x+2)$$

Least Common Denominator

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$$LCD = 2x(x+2)$$

$$\frac{x+2}{x(x+1)} + \frac{1}{2x(x+2)} = \frac{2(x+2)+1}{2x(x+1)} = \frac{2x+5}{2x(x+2)}$$

Least Common Denominator

$$\frac{1}{x^3} - \frac{2}{x^4} + \frac{1}{x^5}$$

Least Common Denominator

$$\frac{1}{x^3} - \frac{2}{x^4} + \frac{1}{x^5}$$

$$LCD = x^5$$

Least Common Denominator

$$\frac{1}{x^3} - \frac{2}{x^4} + \frac{1}{x^5}$$

$$LCD = x^5$$

$$\frac{1}{x^3} - \frac{2}{x^4} + \frac{1}{x^5} = \frac{x^2 - 2x + 1}{x^5} = \frac{(x-1)^2}{x^5}$$

Inverse Problem: "Decombining" Fractions

Often it is useful to make from one fraction two fractions.

Inverse Problem: "Decombining" Fractions

Problem: Find infinite sum:

$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \ldots$$

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$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \ldots$$

Decombine

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{(x+1)}$$

Inverse Problem: "Decombining" Fractions

Problem: Find infinite sum:

$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \ldots$$

Decombine

$$\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{(x+1)}$$

$$\frac{1}{(x+1)(x+2)} = \frac{1}{(x+1)} - \frac{1}{(x+2)}$$

$$\frac{1}{(x+2)(x+3)} = \frac{1}{(x+2)} - \frac{1}{(x+3)}$$

Inverse Problem: "Decombining" Fractions

Problem: Find infinite sum:

$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \ldots =$$

$$\left(\frac{1}{x} - \frac{1}{(x+1)}\right) + \left(\frac{1}{(x+1)} - \frac{1}{(x+2)}\right) + \left(\frac{1}{(x+2)} - \frac{1}{(x+3)}\right) + \ldots$$

Inverse Problem: "Decombining" Fractions

Problem: Find infinite sum:

$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \ldots =$$

$$\frac{1}{x}$$

$$+\left(-\frac{1}{(x+1)} + \frac{1}{(x+1)}\right)+\left(-\frac{1}{(x+2)} + \frac{1}{(x+2)}\right)+\left(-\frac{1}{(x+3)} + \frac{1}{(x+3)}\right)+\ldots$$

Inverse Problem: "Decombining" Fractions

Problem: Find infinite sum:

$$\frac{1}{x(x+1)} + \frac{1}{(x+1)(x+2)} + \frac{1}{(x+2)(x+3)} + \frac{1}{(x+3)(x+4)} + \ldots =$$

$$\frac{1}{x}$$

$$+\left(-\frac{1}{(x+1)} + \frac{1}{(x+1)}\right)+\left(-\frac{1}{(x+2)} + \frac{1}{(x+2)}\right)+\left(-\frac{1}{(x+3)} + \frac{1}{(x+3)}\right)+\ldots$$

$$= \frac{1}{x}$$

QUIZ

Add or subtract fractions:

(1) $\dfrac{x+y}{3xy} + \dfrac{1}{3x}$

(2) $\dfrac{x+y}{x-y} - \dfrac{x-y}{x+y}$

(3) $\dfrac{1}{x^2(x-1)} + \dfrac{x}{(x+1)(x-1)}$

Simplify fraction:

(4) $\dfrac{a^2b^3c}{(a+b)^2bc}$

(5) $\dfrac{x(x^2-1)}{(x+1)(x-1)}$

Write as a sum of two fractions:

(6) $\dfrac{x+2}{(x+1)^2} = \dfrac{\ldots}{(x+1)^2} + \dfrac{\ldots}{(x+1)}$