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Aleksander Zujev  
De Haas-van Alphen Effect talk 2-15-2008

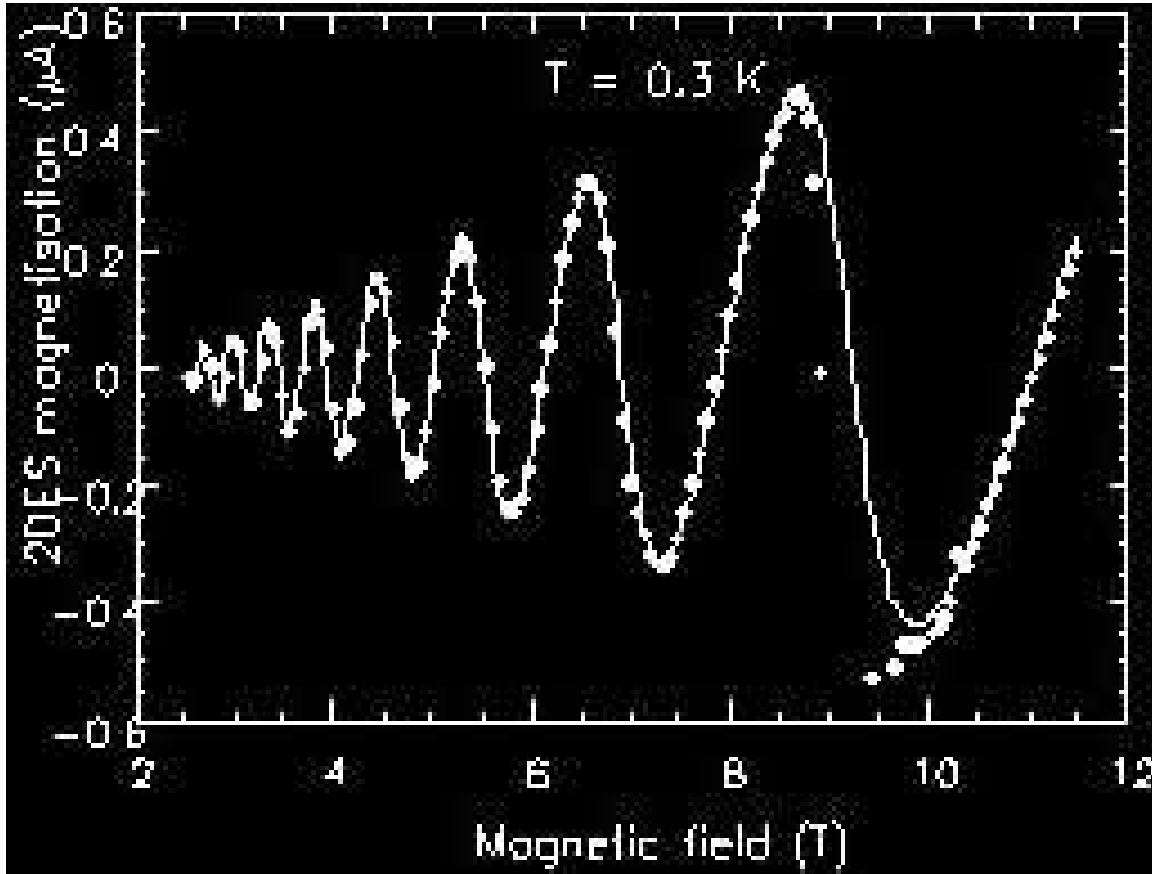
De Haas-van Alphen Effect

- De Haas-van Alphen Effect
- History
- Explanation
- Related Effects
- Applications
- Recent Developments

# What is De Haas-van Alphen Effect

De Haas-van Alphen Effect is the oscillation of the magnetic moment of a metal as a function of the static magnetic field intensity.

Observed best: Pure specimens, low  $T$ , high  $\vec{B}$ .



Magnetisation of a 10-layer 2 dimensional electron system (sample grown at Cambridge University), measured in a collaboration with partners at Exeter University. The solid line is a theoretical fit to the data (solid points).

## ● History

- Discovered by de Haas and van Alphen in 1930. Measured magnetization of bismuth in high magnetic fields at 14.2 K.
- Explained by Onsager in 1952.
- Predicted by Landau in 1930; but he thought magnetic field uniform enough couldn't be achieved.

## • Explanation (simplified)

Free electrons in 2-D (xy plane) in magnetic field  $\vec{B} = B\hat{z}$ .

Energies quantized:

$$E_i = (i + \frac{1}{2})\hbar\omega_c,$$

$$\omega_c = eB/mc$$

Occupation of each level  $D = \rho B$ , where  $\rho = \text{const}$  ( $= eL^2/2\pi\hbar c$ ).

Number of fully occupied levels  $s = \lfloor \frac{N}{D} \rfloor = \lfloor \frac{N}{\rho B} \rfloor$

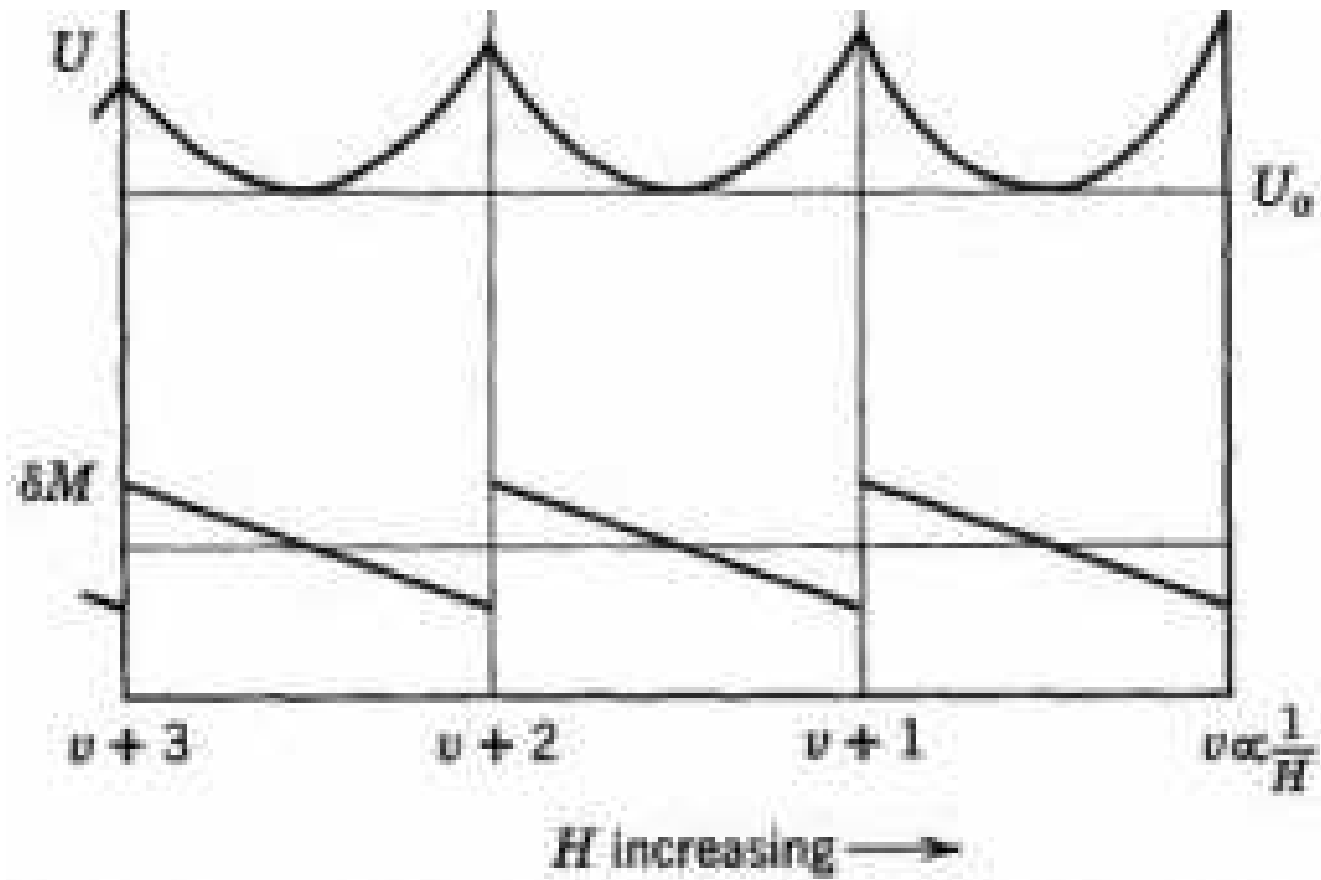
Total energy

$$U = \sum_{i=1}^s D(i - \frac{1}{2})\hbar\omega_c + (s + \frac{1}{2})(N - sD) = \left[ \frac{1}{2}Ds^2 + (s + \frac{1}{2})(N - sD) \right] \hbar\omega_c$$

Floor function: periodic in  $1/B$ .

Magnetization  $M = -\frac{\partial U}{\partial B}$

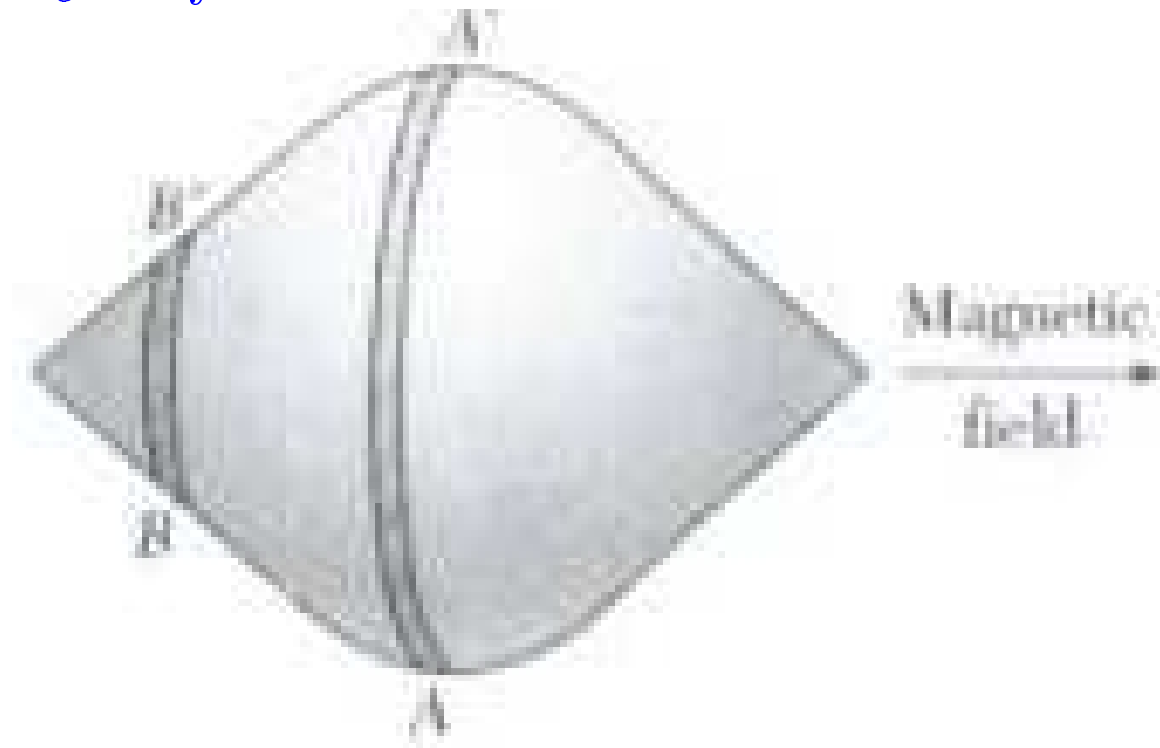
Picture from Kittel:



Onsager: Period in  $1/B$

$$\Delta \left( \frac{1}{B} \right) = \frac{2\pi e}{\hbar c A_e},$$

$A_e$  - any extremal cross-section of Fermi surface  $\perp \vec{B}$ .



Real derivation may be found in textbooks:

Kittel, Introduction to Solid State Physics

Kittel, Quantum Theory of Solids

Ashcroft and Mermin, Solid State Physics



## ● Related Phenomena

**Shubnikov-de Haas (SdH) effect:**

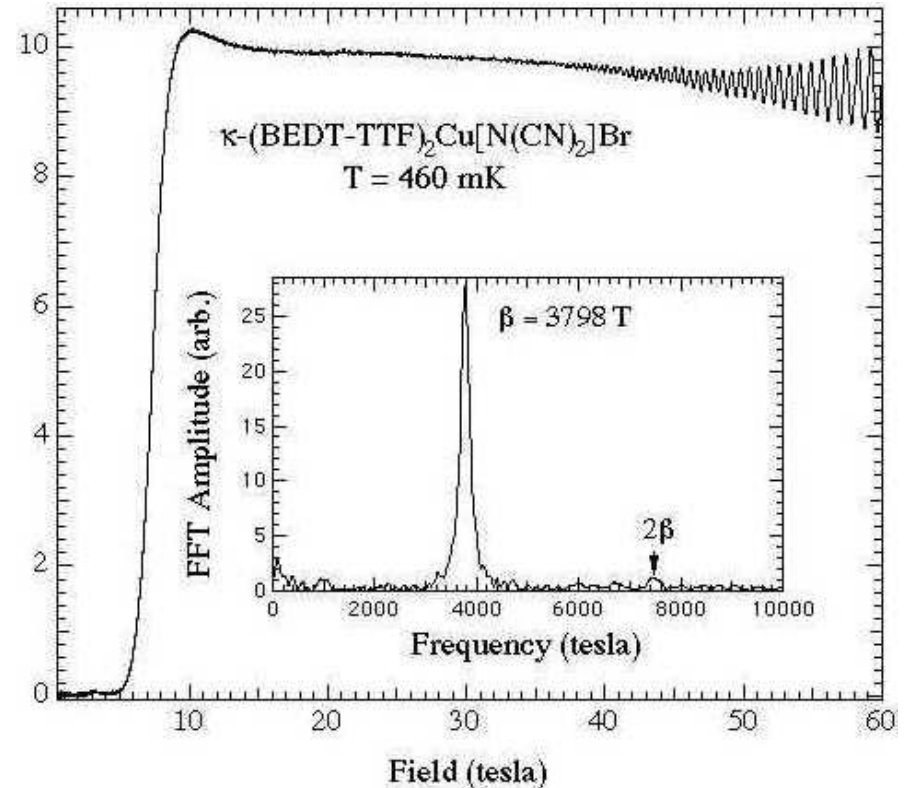
Magnetoresistance oscillations periodic in  $1/B$ .

Often used to determine the effective mass of charge carriers (electrons and electron holes), allowing investigators to distinguish among majority and minority carrier populations.

Chuck Mielke, Fedor Balakirev (LANL):

At the NHMFL-Los Alamos we routinely measure the Fermi surfaces of various compounds by use of the SdH effect.

Our techniques usually employ an rf lock-in amplifier, low temperatures, and a milli-second length pulsed field.



## • Related Phenomena

**Magnetostriction** - Dependence of sample size on magnetic field.

Picture: Quantum Oscillatory Magnetostriction of Heavy Fermion Material  $\text{CeB}_6$ .  
(R. W. Hill, T. J. B. M. Janssen, P. J. Meeson, M. N. Cuthbert, M. Springford and A. L. Wasserman, 1996)

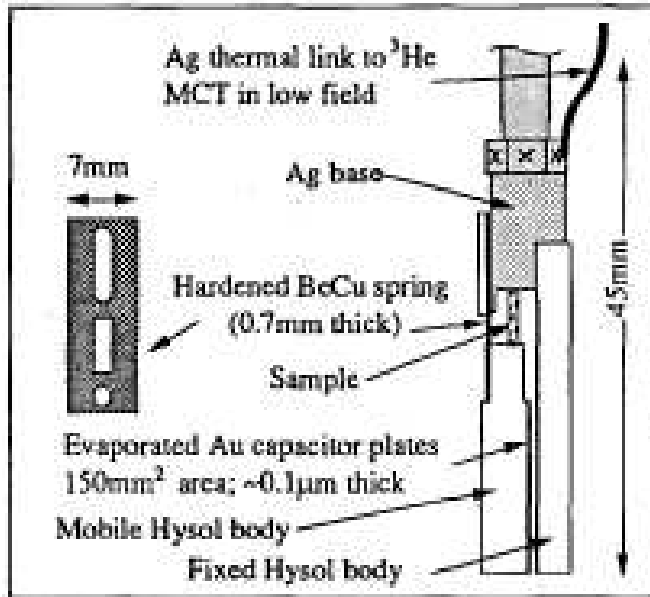


Figure 1: Schematic of the dilatometer.

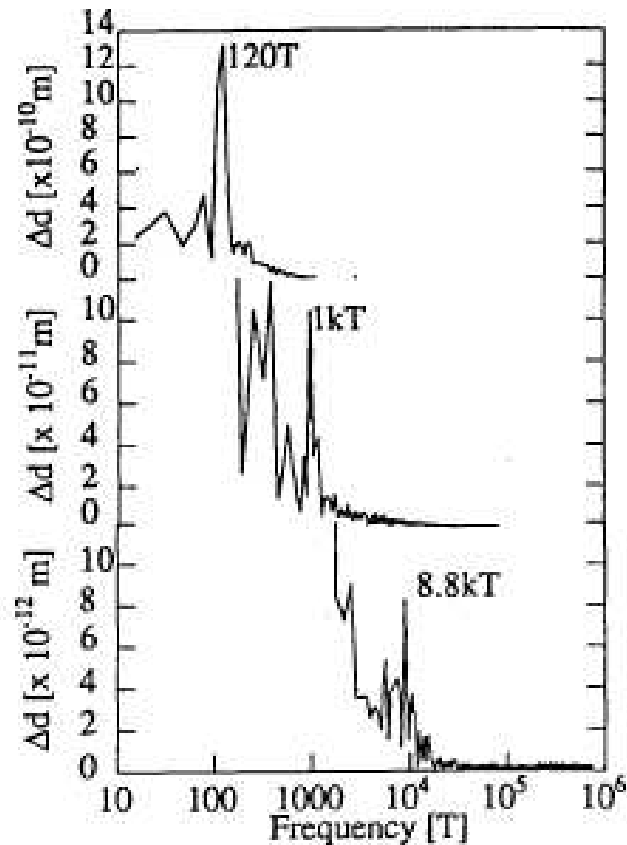


Figure 2: Fast Fourier transform of the QOM signals.  $\Delta d$  is the change in capacitor plate spacing.

# ● Application: Use of de Haas-van Alphen Effect and Related Effects

Measuring the Fermi Surface.

David Shoenberg - developed much of techniques.

Major Techniques:

1. Measure angular position of a sample: in  $\vec{B}$  it experiences torque  $\sim M$ .
2. Measure voltage induced in pickup coil: in changing  $B$  voltage  $\sim \left(\frac{dM}{dt}\right) = \left(\frac{dM}{dB}\right) \left(\frac{dB}{dt}\right)$ .

# • Application

arXiv:cond-mat/0612145

de Haas-van Alphen effect investigations of the electronic structure of pure and aluminum-doped MgB<sub>2</sub>

Authors: A. Carrington, E.A. Yelland, J.D. Fletcher, J.R. Cooper

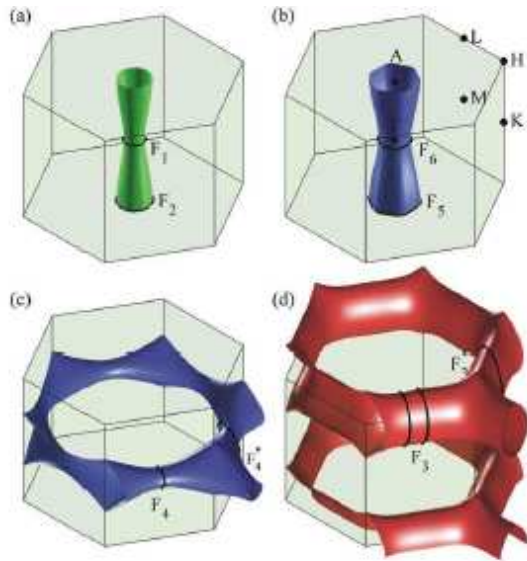


Fig. 1. (color) Calculated Fermi surface of MgB<sub>2</sub>, with possible dHvA extremal orbits (for frequencies <10 kT) indicated. Panels (a)-(d) show the  $\sigma$ -light hole,  $\sigma$ -heavy hole,  $\pi$ -hole and  $\pi$ -electron bands respectively.

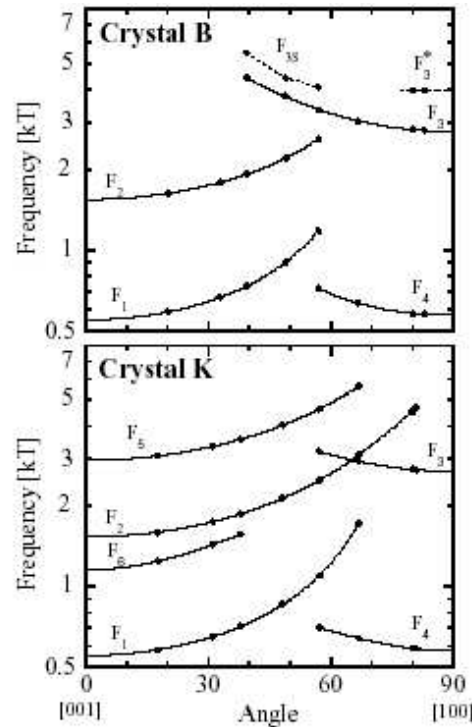


Fig. 2. Observed frequencies versus field angle as the samples were rotated from  $H||c$  to (approximately)  $H||a$ . The solid lines are polynomial fits, and the dotted lines are guides to the eye.

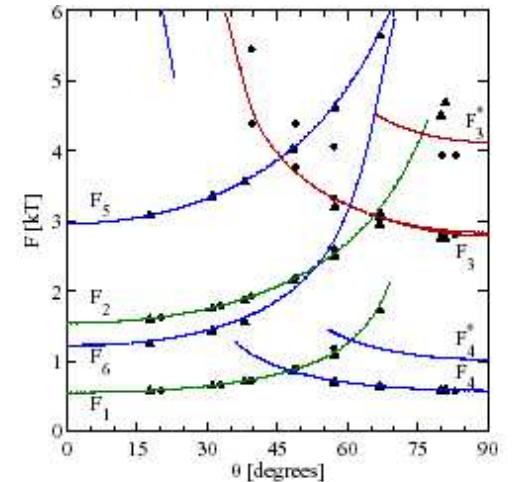


Fig. 3. (color) Comparison of the observed dHvA frequencies with extremal FS areas extracted from the calculated band structure ( $\blacktriangle$ —crystal K,  $\bullet$ —crystal B). Calculated frequencies are for an in-plane angle  $\phi = 12^\circ$ , to match crystal B.

# • Application

Phys. Rev. Lett. 36, 422 - 425 (1976)

Fermi-Surface Measurements in Normal and Superconducting 2H-NbSe<sub>2</sub>  
 John E. Graebner and M. Robbins, Bell Laboratories, Murray Hill,  
 New Jersey 07974

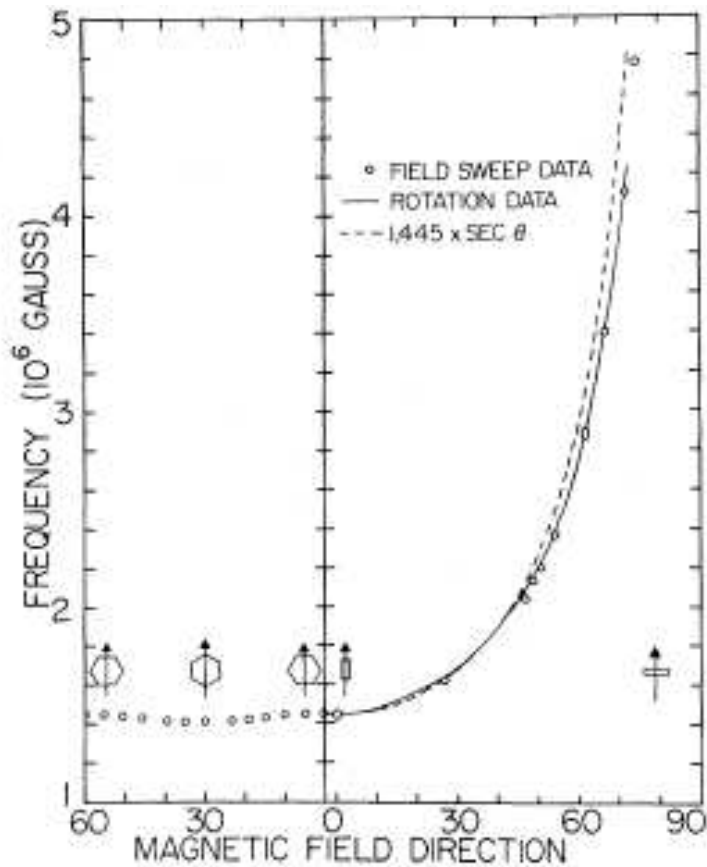


FIG. 2. Frequency versus magnetic-field orientation.

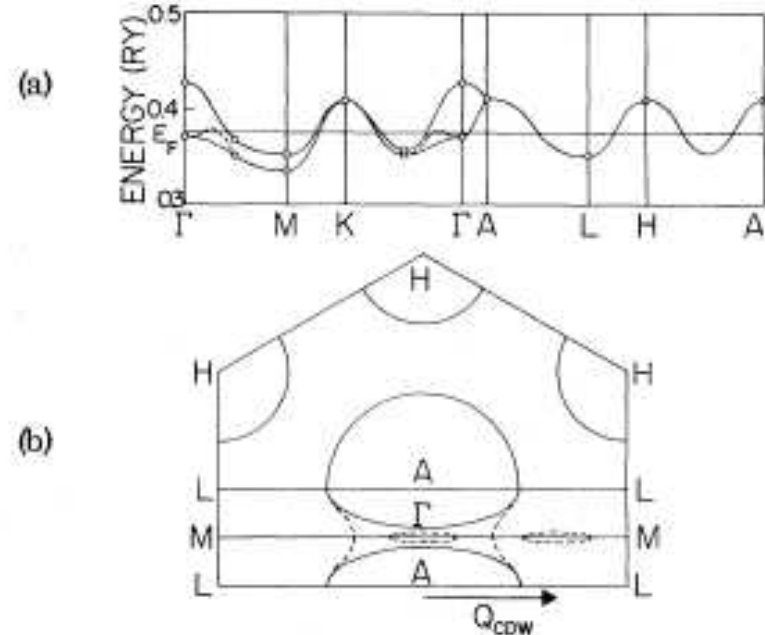


FIG. 3. (a) Solid lines, APW-tight-binding energy bands (Ref. 8) near the Fermi level; dashed lines, proposed modifications to explain the present data. (b) Corresponding Fermi surface and proposed modifications. Translation of the pancake at  $\Gamma$  by  $\vec{Q}_{CDW}$  is also shown.

## ● Application

Phys. Rev. Lett. 80, 3360 - 3363 (1998)

de Haas-van Alphen Effect in Anisotropic Superconductors in Magnetic Fields Well Below  $H_{c2}$

L. P. Gor'kov and J. R. Schrieffer

We develop a quasiclassical approach to the energy spectrum of an anisotropic superconductor in a magnetic field,  $B$ , such that  $H_{c1} \ll B \ll H_{c2}$ . Low temperature de Haas-van Alphen oscillations are considered for two cases: (1) the extremal electron orbit may coincide with a symmetry line and (2) the orbit crosses points where the superconducting order parameter has zeros. The signal is shown to be small in both cases.

Phys. Rev. B 66, 184516 (2002)

## Theory of the de Haas-van Alphen effect in type-II superconductors Kouji Yasui and Takafumi Kita

Theories of quasiparticle spectra and the de Haas-van Alphen (dHvA) oscillation in type-II superconductors are developed based on the Bogoliubov-de Gennes equations for vortex-lattice states. As the pair potential grows through the superconducting transition, each degenerate Landau level in the normal state splits into quasiparticle bands in the magnetic Brillouin zone. This brings Landau-level broadening, which in turn leads to the extra dHvA oscillation damping in the vortex state. We perform extensive numerical calculations for three-dimensional systems with various gap structures. It is thereby shown that (i) this Landau-level broadening is directly connected with the average gap at  $H=0$  along each Fermi-surface orbit perpendicular to the field  $H$ , (ii) the extra dHvA oscillation attenuation is caused by the broadening around each extremal orbit. These results imply that the dHvA experiment can be a unique probe to detect band- and/or angle-dependent gap amplitudes. We derive an analytic expression for the extra damping based on the second-order perturbation with respect to the pair potential for the Luttinger-Ward thermodynamic potential. This formula reproduces all our numerical results excellently, and is used to estimate band-specific gap amplitudes from available data on NbSe<sub>2</sub>, Nb<sub>3</sub>Sn, and YNi<sub>2</sub>B<sub>2</sub>C. The obtained value for YNi<sub>2</sub>B<sub>2</sub>C is fairly different from the one through a specific-heat measurement, indicating presence of gap anisotropy.

Rev. Mod. Phys. 73, 867 (2001)

Vortex states and quantum magnetic oscillations in conventional type-II superconductors

Tsofar Maniv, Vladimir Zhuravlev, Israel Vagner, and Peter Wyder

The theory of pure type-II superconductors at high magnetic fields and low temperatures has recently attracted much attention due to the discovery of de Haasvan Alphen oscillations deep in the vortex state. In this article the authors review the state of the art in this rapidly growing new field of research...

(45 pages)



Phys. Rev. B 72, 212504 (2005)

de Haas-van Alphen effect in metals without an inversion center

V. P. Mineev and K. V. Samokhin

We show how the de Haas-van Alphen effect can be used to directly measure the magnitude of spin-orbit coupling in noncentrosymmetric metals, such as  $\text{CePt}_3\text{Si}$  and  $\text{LaPt}_3\text{Si}$ .

● [arXiv:0707.3785](https://arxiv.org/abs/0707.3785) High Energy Physics - Phenomenology

Title: Color superconducting matter in a magnetic field

Authors: Kenji Fukushima, Harmen J. Warringa

We investigate the effect of a magnetic field on cold dense three-flavor quark matter using an effective model with four-Fermi interactions with electric and color neutrality taken into account. The gap parameters  $\Delta_1$ ,  $\Delta_2$ , and  $\Delta_3$  representing respectively the predominant pairing between down and strange (d-s) quarks, strange and up (s-u) quarks, and up and down (u-d) quarks, show the de Haas-van Alphen effect, i.e. oscillatory behavior as a function of the modified magnetic field  $B$  that can penetrate the color superconducting medium...