UCDAVIS

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Fractional Quantum Hall Effect and Quantum Computer

Outline

- Hall effect
- Quantum Hall effect
- Fractional Quantum Hall effect
- Fractional Quantum Hall effect and Quantum Computer

Hall effect

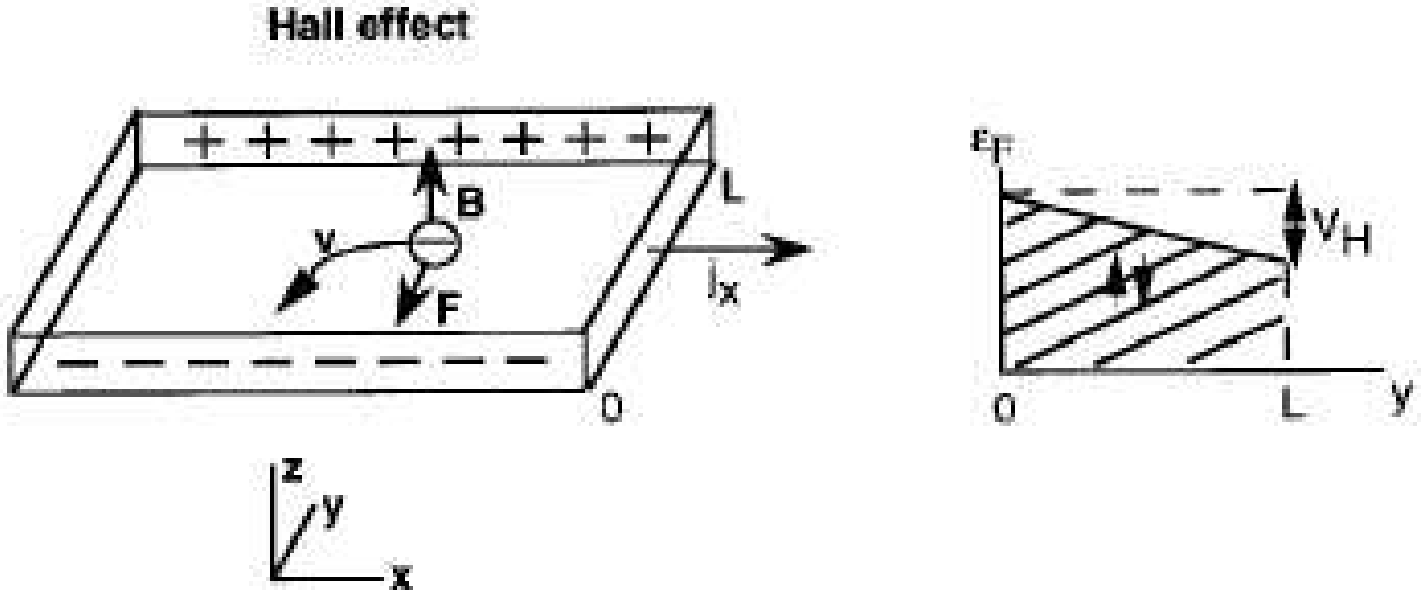
Electric current in magnetic field $\vec{B} \perp \vec{j}$

$\Rightarrow \vec{E}_H \perp \vec{j}, \vec{B}$

$E_H = \rho_H j$

$\rho_H = R_0 B$

R_0 - Hall coefficient. Easy to show: $R_0 = \frac{1}{en}$



Math

Electron moving in $\vec{B} \perp \vec{v}$.
Equilibrium between the forces:

$$evB = eE$$

$$j = nev$$

$$\Rightarrow E = j\frac{1}{ne}B$$

$$\rho_H = \frac{E}{j} = \frac{1}{ne}B$$

Resistivity tensor (2D, $\vec{B} \parallel \hat{z}$):

$$\rho = \begin{bmatrix} \rho_0 & \rho_H \\ -\rho_H & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & B/ne \\ -B/ne & \rho_0 \end{bmatrix}$$

Quantum Hall Effect

Ordinary Hall Effect in 2-D:
$\vec{B} \parallel \hat{z}$
Resistivity tensor

$$\rho = \begin{bmatrix} \rho_0 & \rho_H \\ -\rho_H & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & B/ne \\ -B/ne & \rho_0 \end{bmatrix}$$

Quantum Hall Effect

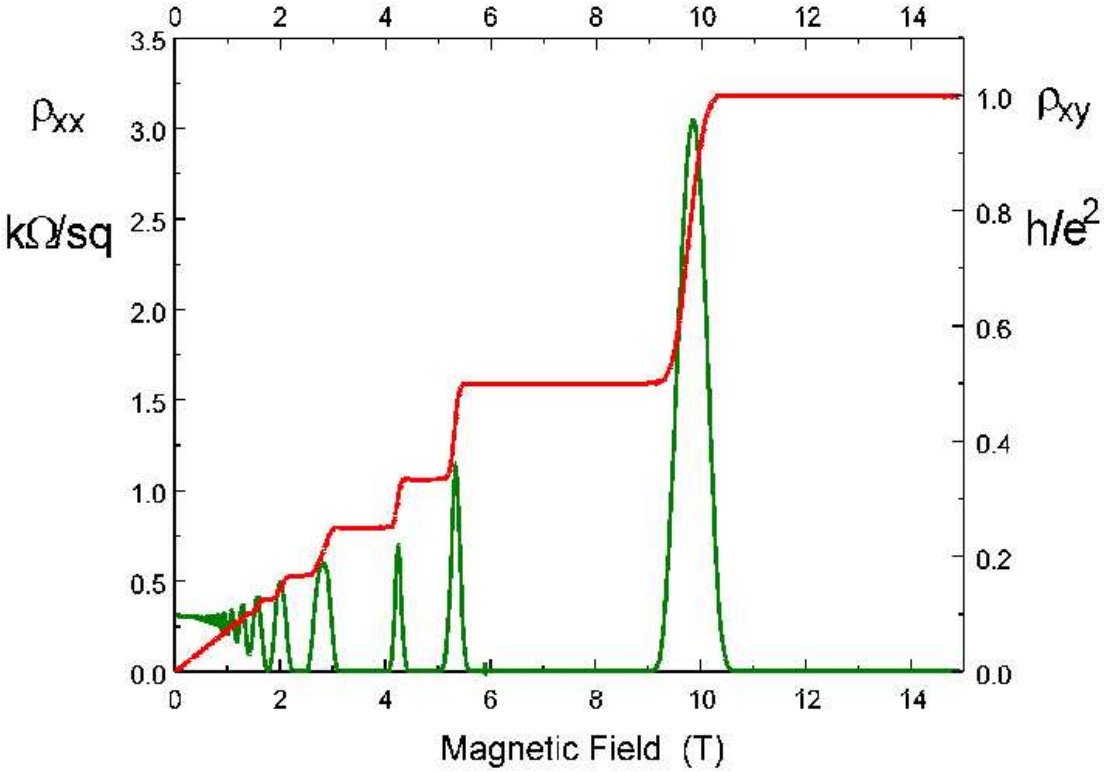
Quantum Hall Effect in 2-D:
$\vec{B} \parallel \hat{z}$
Resistivity tensor

$$\rho = \begin{bmatrix} \rho_0 & \rho_H \\ -\rho_H & \rho_0 \end{bmatrix} = \begin{bmatrix} \rho_0 & h/ie^2 \\ -h/ie^2 & \rho_0 \end{bmatrix}$$

i - an integer.
$\rho_H = h/ie^2 \quad \equiv \quad \sigma_H = ie^2/h$ - quantized.

Quantum Hall Effect

Integer quantum Hall effect in a GaAs-GaAlAs heterojunction, T =30mK. Also diagonal component of resistivity - shows regions of zero resistance corresponding to each QHE plateau.



A Derivation

A plane: $0 < x < L$, $0 < y < W$.
Landau gauge: $A_x = -yB$, $A_y = 0$.
S.E.:

$$\frac{\hbar^2}{2m}\left[\left(-i\frac{\partial}{\partial x} - \frac{eB}{\hbar}y\right)^2 - \frac{\partial^2}{\partial y^2}\right]\psi = E\psi$$

Trying $\psi = e^{ikx}\phi(y)$:

$$\frac{h\omega_c}{2}\left[-l^2\frac{\partial^2}{\partial y^2} + \left(\frac{y}{l} - lk\right)^2\right]\phi = E\phi$$

where $l \equiv (\hbar/eB)^{1/2}$.
- H.O. centered at $y = l^2k \Rightarrow\ 0 < k < W/l^2$.
Solutions are $\phi_{nk}(y) = H_n(y/l - lk)e^{-(y-l^2k)^2/2l^2}$

$E_{nk} = \hbar\omega_c(n + \frac{1}{2})$ - independent of k.
Periodic B.C. on x: $x = 0 \equiv x = L \Rightarrow k = 2\pi p/L$
$\Rightarrow$ number of states in Landau level $LW/2\pi l^2$, or per unit area $n_B = 1/2\pi l^2 = eB/h$.
If every occupied level is full, then "filling factor"

$$\nu = \frac{n}{n_B}$$

- integer, or

$$n = \nu n_B = \nu \frac{eB}{h}$$

$$\rho_H = \frac{B}{ne} = \frac{h}{\nu e^2}$$

Fractional Quantum Hall Effect

As in the integer quantum Hall effect, a series of plateaus forms in the Hall resistance. Each particular values of magnetic field corresponds to a filling factor (the ratio of electrons to magnetic flux quanta)

$$\nu = \frac{p}{q}$$

where p and q are integers with no common factors.
q - odd
except

$$\nu = \frac{5}{2}, \frac{7}{2}$$

Principal series

$$\frac{1}{3}, \frac{2}{5}, \frac{3}{7} \cdots$$

$$\frac{2}{3}, \frac{3}{5}, \frac{4}{7} \cdots$$

Theories

- Fractionally-charged quasiparticles (Laughlin): hides interactions by constructing a set of quasiparticles with charge $e^* = \frac{e}{q}$.
- Composite Fermions (Jain, and Halperin, Lee and Read): to hide the interactions, it attaches two (or, in general, an even number) flux quanta $\frac{h}{e}$ to each electron, forming integer-charged quasiparticles called composite fermions. The fractional states are mapped to the integer QHE. This makes electrons at a filling factor 1/3, for example, behave in the same way as at filing factor 1. A remarkable result is that filling factor 1/2 corresponds to zero magnetic field. Experiments support this.

Anyons

Fermi-Dirac, Bose-Einstein:

$$|\psi_1\psi_2> = \pm|\psi_2\psi_1>$$

Anyons:

$$|\psi_1\psi_2> = e^{i\theta}|\psi_2\psi_1>$$

Fractional QHE: Anyons

Laughlin: $\frac{p}{q}$ QHE:
quasiparticle excitations: anyons:
charge $\frac{e}{q}$
statistical angle $\theta = \frac{\pi}{q}$

Fractional QHE: Moore-Read Pfaffian states

For reasons outside of this review (read: too complicated for me) $\frac{p}{q} = \frac{5}{2}$ QHE is suitable for Quantum Computation, because its anyons have non-Abelian statistics.

Another possibility $\frac{p}{q} = \frac{12}{5}$ - supposedly even better, have braiding statistics that allow universal topological quantum computation.

Gregory Moore, Nicholas Read, Nonabelions in the fractional quantum Hall effect, Nuclear Physics B, 360, Pages 362-396 (1991).

- Excitations: quasiparticles with Non-Abelian statistics.

Braids

Quasiparticles worldlines: Braids. Braid group $\mathcal{B}_3$:

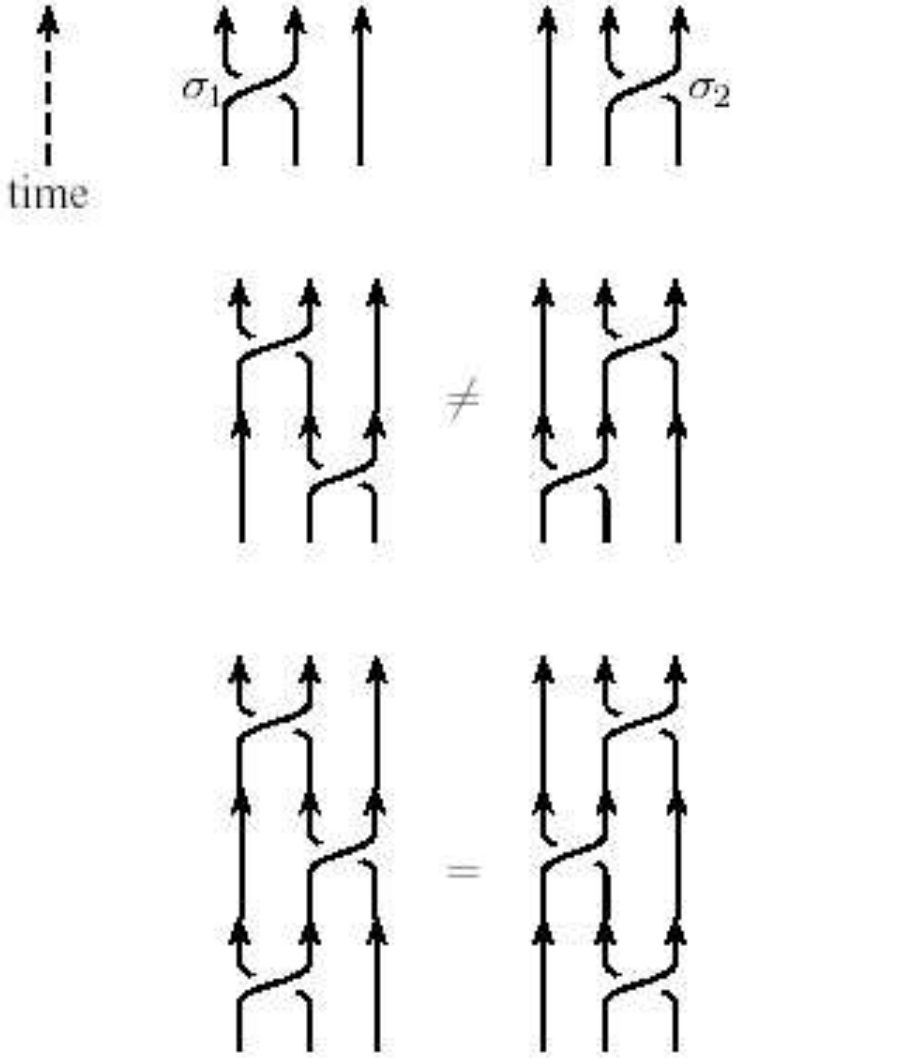


FIG. 1 **Top:** The two elementary braid operations σ_1 and σ_2 on three particles. **Middle:** Here we show $\sigma_2\sigma_1 \neq \sigma_1\sigma_2$, hence the braid group is Non-Abelian. **Bottom:** The braid relation (Eq. 3) $\sigma_i\sigma_{i+1}\sigma_i = \sigma_{i+1}\sigma_i\sigma_{i+1}$.

Requirements for quasiparticles to follow Non-Abelian statistics

- 1. N-quasiparticle GS is degenerate. For this, quasiparticles must be well separated.
- 2. Interchange of quasiparticles: U transformation, whose non-Abelian part is determined only by the topology of braid, and non-topological part is Abelian.
- 3. The only way to make U transform on degenerate GS space is by braiding.

Topological Quantum Computer

Alexei Kitaev, "Fault-tolerant quantum computation by anyons," arXiv:quant-ph/9707021, Annals Phys. 303 (2003) 2-30

Reason: Any local perturbation has no nontrivial matrix elements within the ground state subspace. Thus, the system is immune from decoherence (Kitaev, 2003).

Topological Quantum Computer

Sankar Das Sarma, Michael Freedman, and Chetan Nayak, "Topologically Protected Qubits from a Possible Non-Abelian Fractional Quantum Hall State", Phys. Rev. Lett. 94, 166802 (2005).

Chetan Nayak[3], Steven H. Simon, Ady Stern, Michael Freedman, Sankar Das Sarma, "Non-Abelian Anyons and Topological Quantum Computation", http://www.arxiv.org/a 2007

Quantum computation:

Initialize the state of qubits;

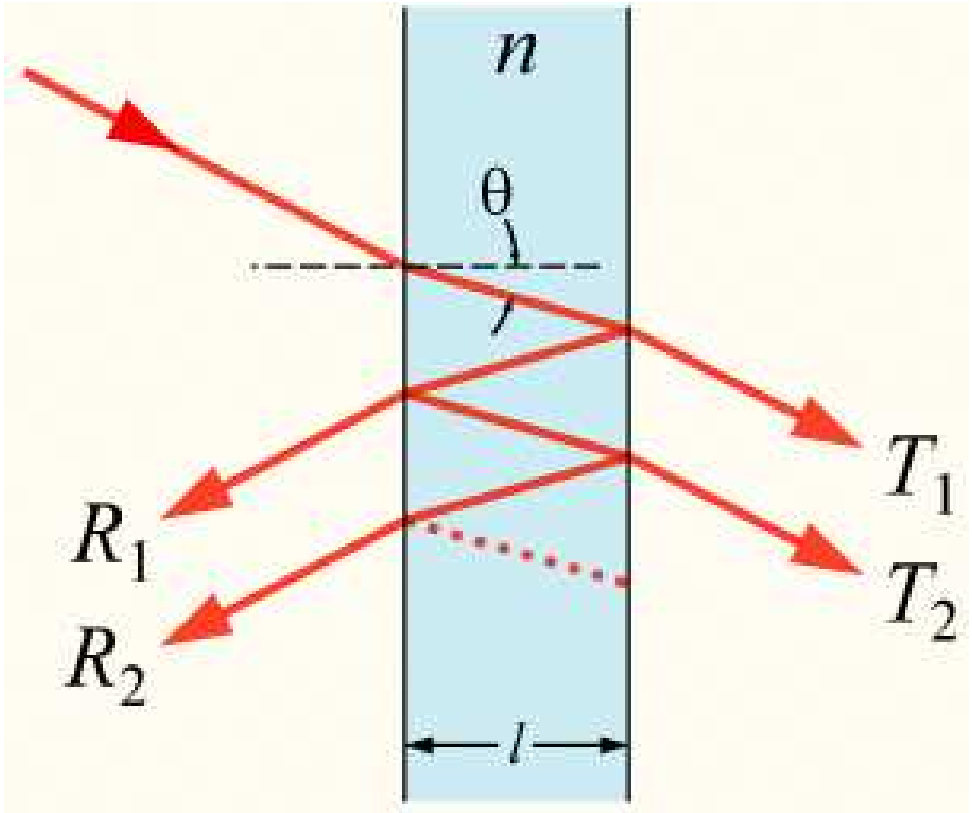
Perform arbitrary controlled unitary operations on the state;

Measure the state of qubits at the end.

Topological Quantum Computer

Review:

Fabry-Perot interferometer



Topological Quantum Computer

Quantum Hall analog of Fabry-Perot interferometer

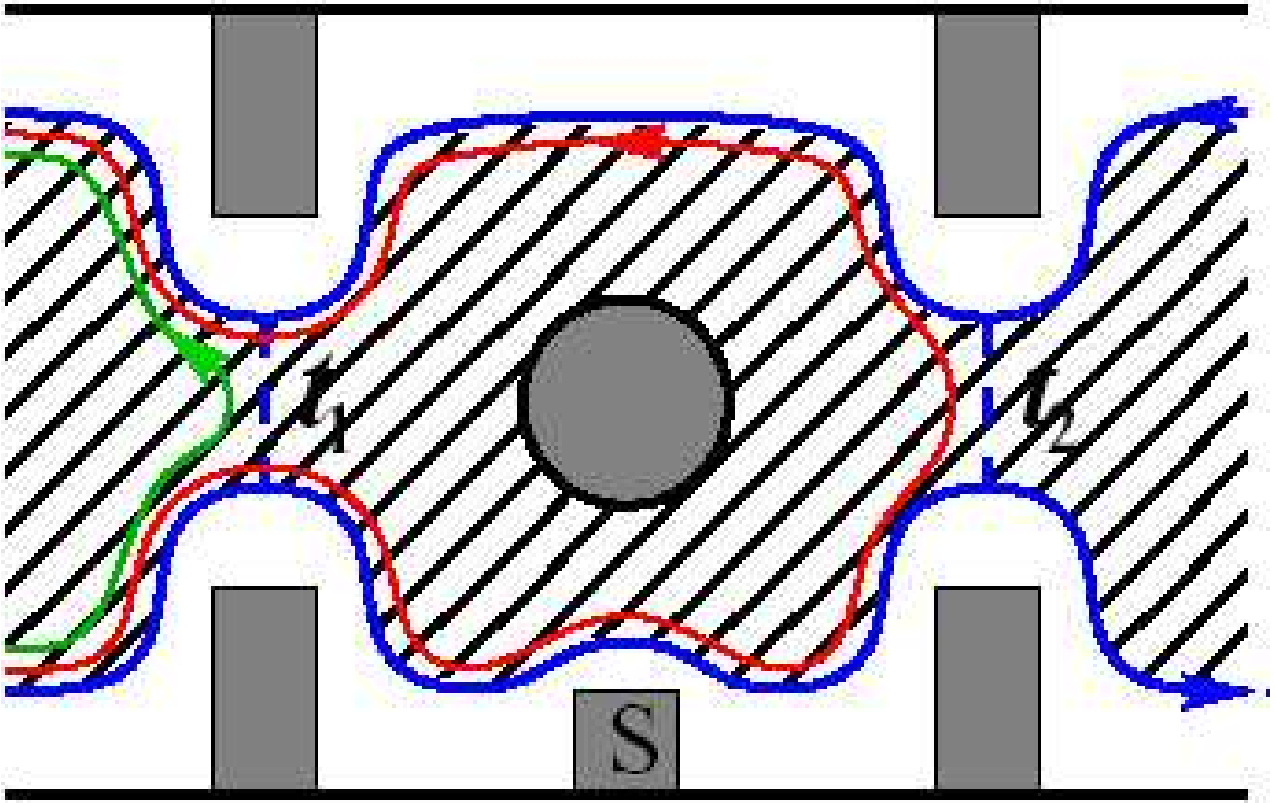


FIG. 2 A quantum Hall analog of a Fabry-Perot interferometer. Quasiparticles can tunnel from one edge to the other at either of two point contacts. To lowest order in the tunneling amplitudes, the backscattering probability, and hence the conductance, is determined by the interference between these two processes. The area in the cell can be varied by means of a side gate S in order to observe an interference pattern.

Topological Quantum Computer

Constructing Qubits

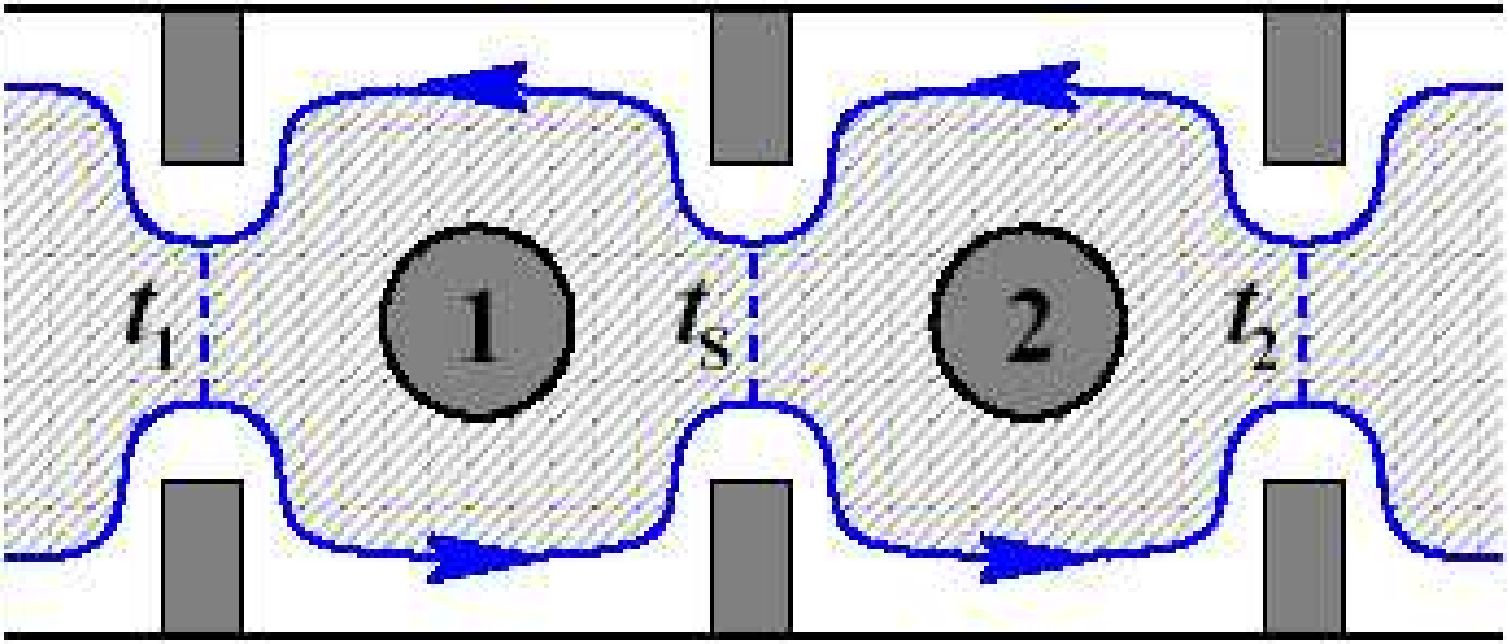


FIG. 3 If a third constriction is added between the other two, the cell is broken into two halves. We suppose that there is one quasiparticle (or any odd number) in each half. These two quasiparticles (labeled 1 and 2) form a qubit which can be read by measuring the conductance of the interferometer if there is no backscattering at the middle constriction. When a single quasiparticle tunnels from one edge to the other at the middle constriction, a σ_x or NOT gate is applied to the qubit.

Alternatives to QHE

Topological phases: possibilities:

- Transition metal oxides

- ultra-cold atoms in optical traps:

Use rotation instead of magnetic field to get analog of QHE.

References

- Non-Abelian Anyons and Topological Quantum Computation Authors: Chetan Nayak, Steven H. Simon, Ady Stern, Michael Freedman, Sankar Das Sarma, arXiv:0707.1889

- Computing with Quantum Knots, Graham P. Collins, Scientific American, April 2006