Introduction to Luttinger Liquid

- What is Fermi Gas
- What is Fermi Liquid
- Fermi Liquid in 1-D: Breakdown
- Introduction of Luttinger Liquid
- Properties of Luttinger Liquid
- Experiments with Luttinger Liquids

What is Fermi Gas

Fermi gas - collection of non-interacting fermions.
T=0. Pauli exclusion principle: 1 e^- / h^3;
n e^- occupy sphere in momentum space

$$2 \cdot \frac{4}{3}\pi p_F^3 \; = \; nh^3$$

$$\frac{8}{3}\pi(\hbar k_F)^3 \; = \; n(2\pi\hbar)^3$$

$$k_F = (3\pi^2 n)^{1/3}$$

$$C_V = \frac{mk_F}{3\hbar^2}k_B^2 T$$

What is Fermi Liquid

Fermi liquid = Fermi gas + Interactions

• Landau Fermi Liquid Theory

Idea: one to one correspondence between eigenstates (ground state and excited states) of noninteracting and interacting system.

Crucial: interactions do not lead to phase transition or symmetry broken ground state.
Particle → Landau quasiparticle

Works Ok. E.g. C_V.
Fermi gas:

$$C_V = \frac{mk_F}{3\hbar^2} k_B^2 T$$

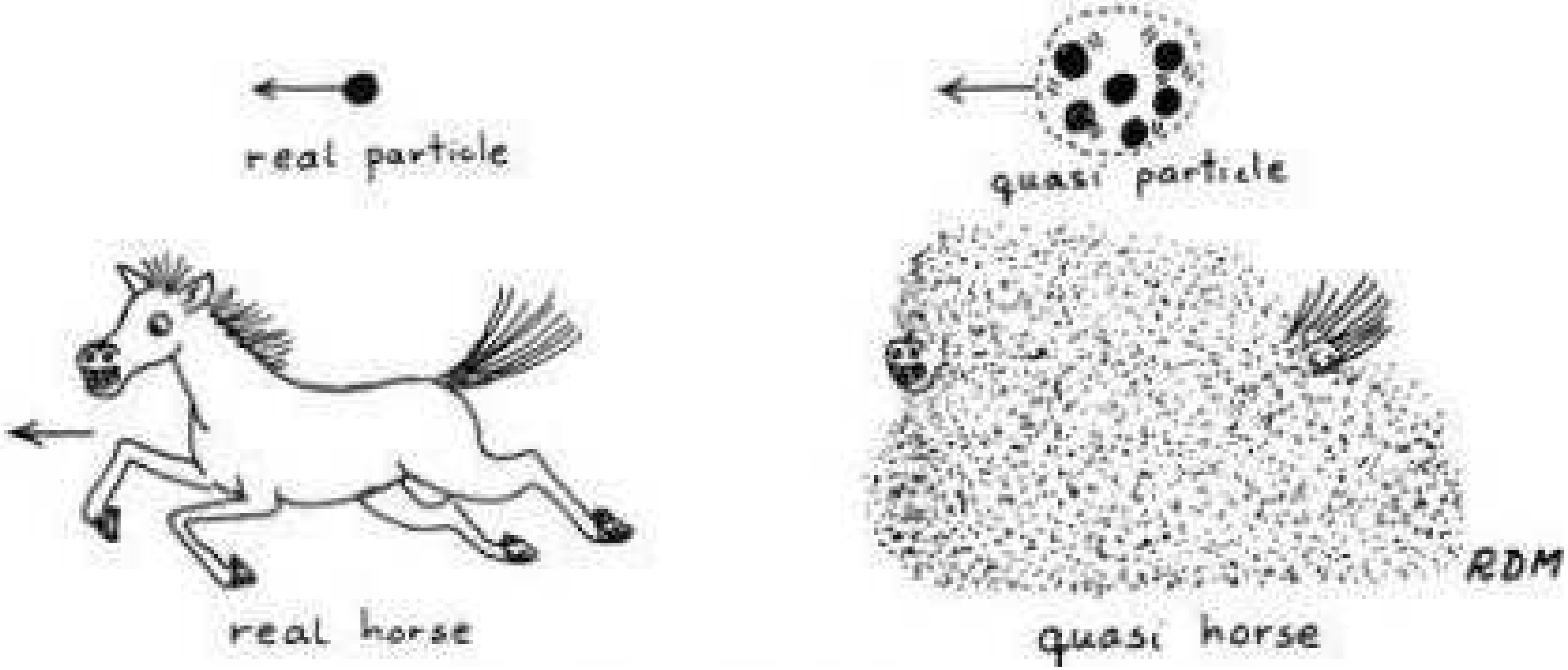
Fermi liquid:

$$C_V = \frac{m^* k_F}{3\hbar^2} k_B^2 T$$

• Landau Fermi Liquid Theory

NOTE: Landau quasiparticles are very similar to real particles.

Quasiparticle = real particle + cloud of other particles = ”clothed” particle.



• Fermi Liquid in 1-D: Breakdown

General Considerations.

Dimensionality plays key role in phase transitions.

How correlation functions decay
Classical Models

	$d = 1$	$d = 2$	$d = 3$
$Ising$	$T_c = 0$	$T_c \neq 0$	$T_c \neq 0$
XY	$T_c = 0$	$Kosterlitz - Thouless$	$T_c \neq 0$
$Heisenberg$	$T_c = 0$	$T_c = 0$	$T_c \neq 0$

Quantum systems - harder to order than classical.
d=1: at T=0 the most order possible is algebraic decay, e.g.

$$\langle n(x) n(0) \rangle \sim \frac{1}{x^a}$$

$$\langle m(x) m(0) \rangle \sim \frac{1}{x^b}$$

• Fermi Liquid in 1-D: Breakdown

1-D: Lifetime of Landau quasiparticle $\rightarrow 0$

The concept of a quasiparticle must be abandoned

• Bosonization

Spinless case.
Left and right mover density operators

$$\rho_L(q) = \sum_{k<0} c_k^\dagger c_{k+q} \quad \rho_R(q) = \sum_{k>0} c_k^\dagger c_{k+q}$$

Boson operators

$$a_q = \sqrt{\frac{2\pi}{qL}}\rho_R(q) \quad a_q^\dagger = \sqrt{\frac{2\pi}{qL}}\rho_R(-q)$$

$$b_q = \sqrt{\frac{2\pi}{qL}}\rho_L(-q) \quad a_q^\dagger = \sqrt{\frac{2\pi}{qL}}\rho_L(q)$$

• Luttinger Liquid Hamiltonian

With Spin.
Charge and spin densities

$$\rho = \rho_{\downarrow} + \rho_{\uparrow} \qquad \sigma = \rho_{\downarrow} - \rho_{\uparrow}$$

$$H = H_0 + H_{int}$$

$$H_0 = \frac{\pi v_F}{2L} \sum_{q \neq 0} [\rho_R(-q)\rho_R(q) + \rho_L(q)\rho_L(-q) + \sigma_R(-q)\sigma_R(q) + \sigma_L(q)\sigma_L(-q)]$$

$$H_{int} = \frac{V}{L} \sum_{q \neq 0} [\rho_R(q) + \rho_L(q)][\rho_R(-q) + \rho_L(-q)]$$

Note - charge and spin operators decoupled ⇒ Spin-charge separation.

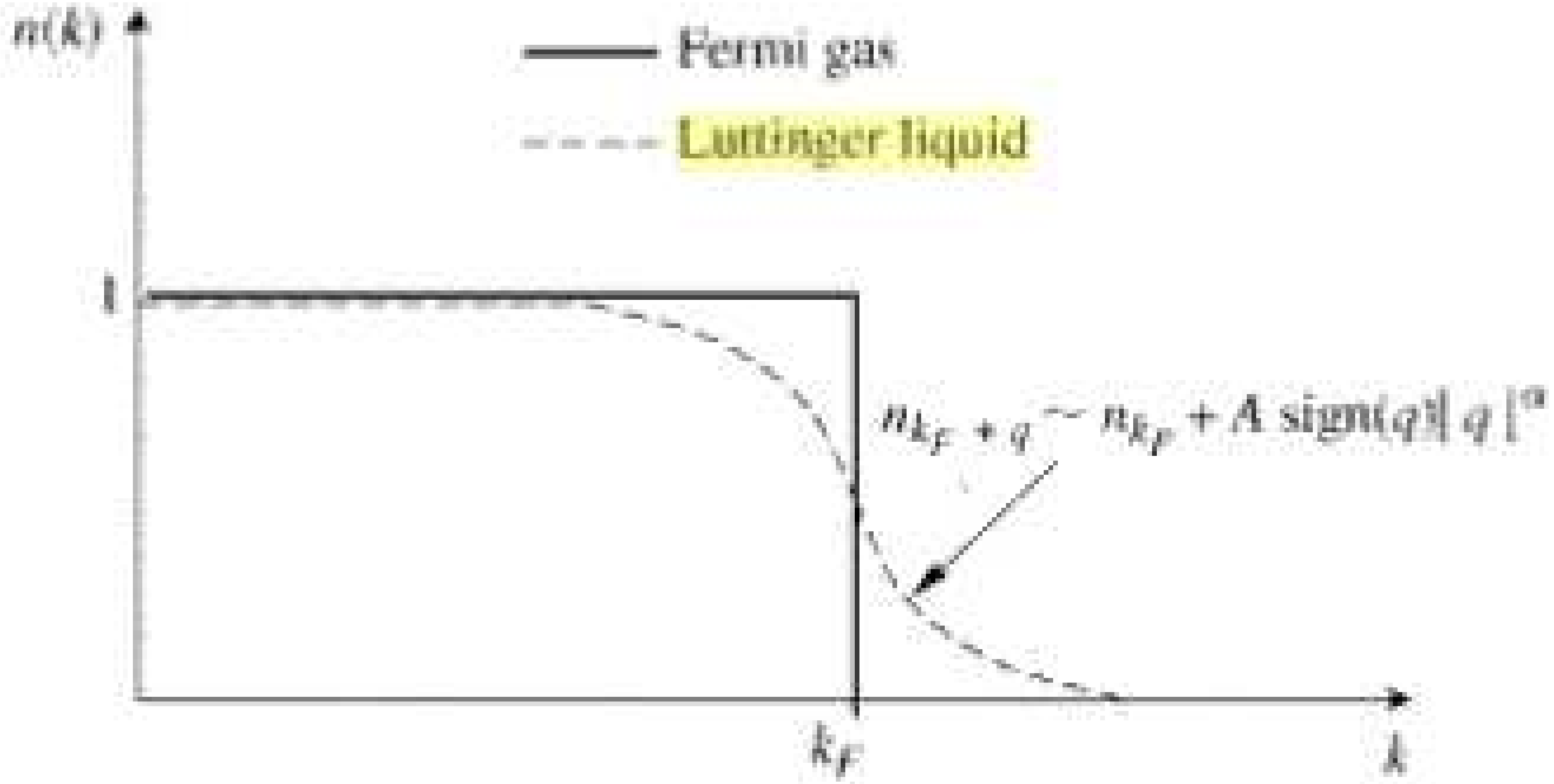
• Luttinger Liquid

Luttinger liquid - ground state of an interacting system which has no quasiparticles similar to that of the non-interacting case.

Instead, it has collective excitations, which bear no resemblance to the original fermions - they are bosonic.

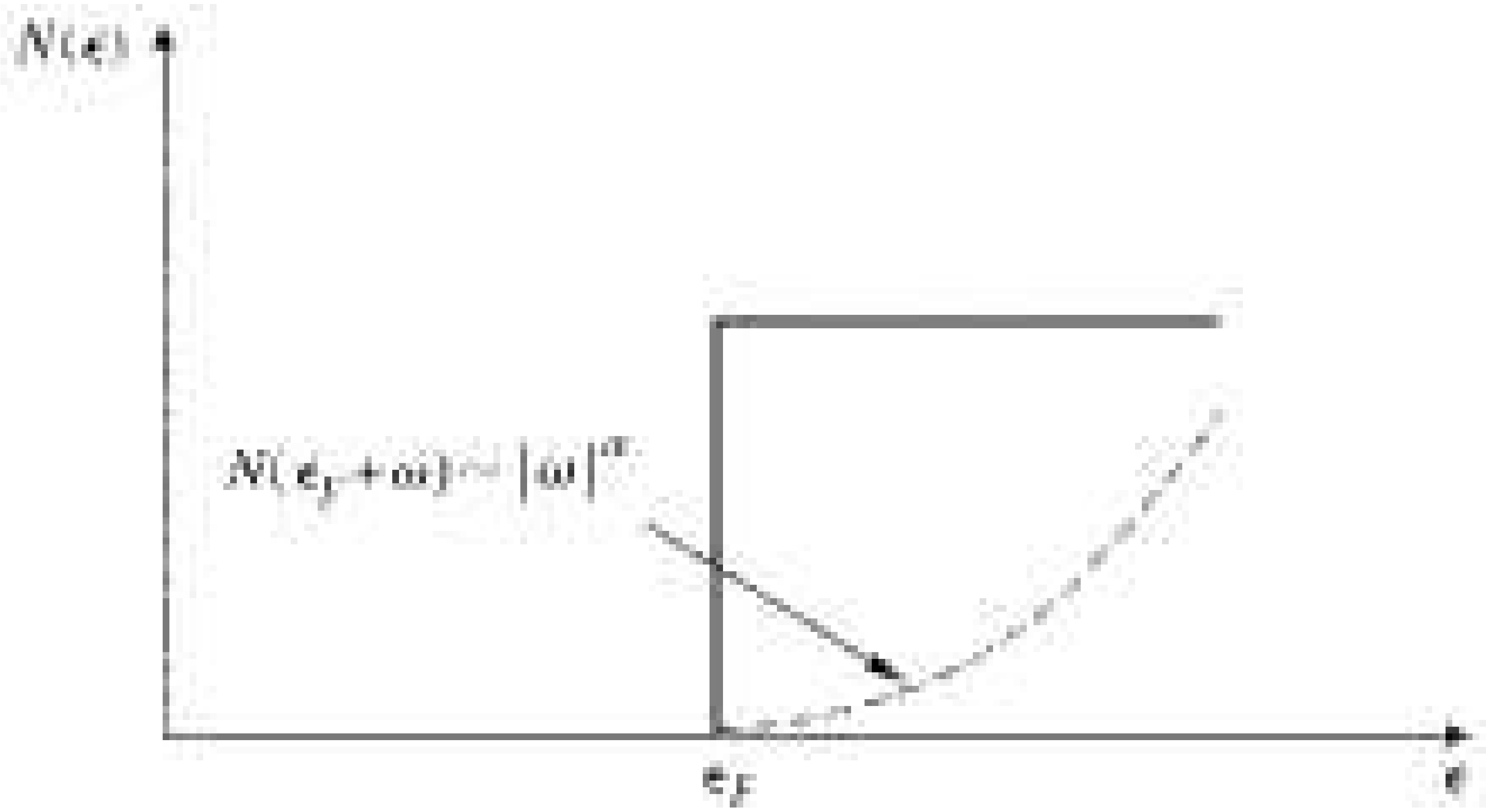
• Properties of Luttinger Liquid

(i) a continuous momentum distribution function n(k) $\sim\ |k - k_F|^{\alpha}$, exponent α - interaction-dependent.



• Properties of Luttinger Liquid

(ia) Tunneling density of states $N(\epsilon) \sim |\epsilon - \epsilon_F|^{\alpha}$



• Properties of Luttinger Liquid

(ii) similar power-law behaviour in all correlation functions, specifially in those for superconducting and spin or charge density wave fluctuations, with universal scaling relations between the different nonuniversal exponents
1) Tomonaga-Luttinger regime (without spin gap):

$$C_{CDW}(k),\ \ C_{SDW}(k)\ \sim |k-2k_F|^{K_\rho}, \quad k \sim 2k_F$$

$$C_{SS}(k),\ \ C_{TS}(k)\ \sim |k|^{1/K_\rho}, \quad k \sim 0$$

2) Luther-Emery regime (with spin gap):

$$C_{CDW}(k)\ \sim |k-2k_F|^{K_\rho}, \quad k \sim 2k_F$$

$$C_{SS}(k)\ \sim |k|^{1/K_\rho - 1}, \quad k \sim 0$$

$$C_{SDW}(k), C_{TS}(k) \quad - \text{no singularity}$$

(SS - singlet superconducting, TS - triplet superconducting)

Power laws are not indepndent of each other. Instead there is a single number K_rho which determines the powers of *all* the decays.

• Properties of Luttinger Liquid

(iii) Finite spin and charge response at small wavevectors, and a finite Drude weight in the conductivity

Kubo formula

$$\sigma(\omega) = \frac{i}{\omega}\left[\frac{D}{\pi} + R_j^R(\omega)\right]$$

$$R_j^R(\omega) = -\frac{i}{L}\int_0^L dx \int_0^\infty dt \langle [j(x,t), j(0,0)]\rangle e^{i\omega t}$$

Drude weight

$$D = \frac{\pi}{2L}\frac{\partial^2 E_0(\Phi)}{\partial \Phi^2}\bigg|_{\Phi=0} = 2v_{J\rho}$$

(E_0 - ground state energy, Φ - applied flux)

• Properties of Luttinger Liquid

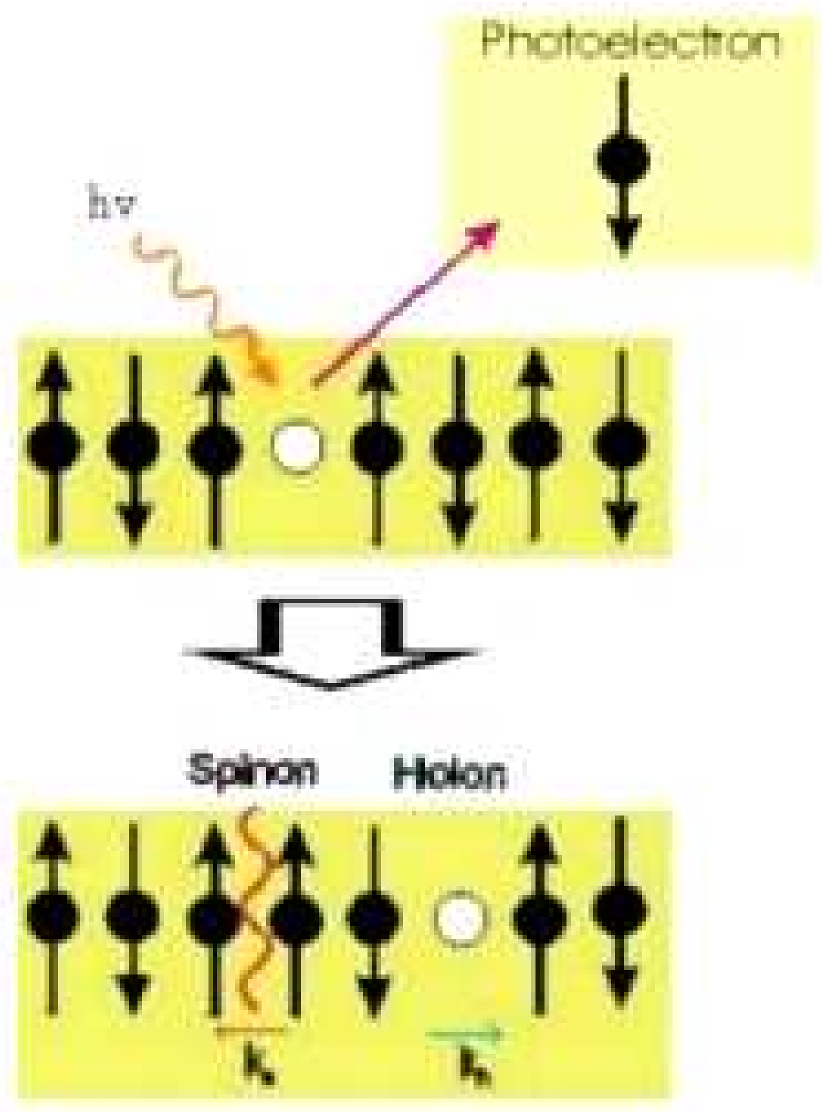
(iv) Spin-charge separation.

$$H = H_\rho + H_\sigma$$

- separation of dynamics of spin and charge. $v_\rho \neq v_\sigma$ - charge and spin oscillations propagate with different velocities.

• Properties of Luttinger Liquid

Spin-charge separation.



• Properties of Luttinger Liquid

(v) Persistent currents quantized in units of $2k_F$.
(? $2ev_F/L$)

• Experiments with Luttinger Liquids

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* Carbon Nanotubes

Tunneling: power law tunneling DOS.

• Experiments with Luttinger Liquids

* Semiconductor wires
Tunneling experiments between two wires show spin-charge separation.

• Experiments with Luttinger Liquids

* Quasi 1D Materials

Some crystals: large anisotropy. Conduction large along one direction, small two others. E.g. Bechgaard salts. Optical response: power law.

- Experiments with Luttinger Liquids

* Edge states in the fractional Quantum Hall Effect.

References

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[2] H.J. Schulz, G. Cuniberti, P. Pieri, Fermi liquids and Luttinger liquids, arXiv:cond-mat/9807366v2

[3] Kazuhiro Sano and Yoshiaki Ono, Electronic Structure of One-Dimensional Extended Hubbard Model, JPSJ 63, No 4, 1250-1253 (1994)