UCDAVIS

Aleksander Zujev
Advisor: Professor Scalettar

Finite size scaling: How-to Introduction using Ising model

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- 1. Scaling hypothesis
- 2. Finding a
- 3. Finding T_c
- 4. Finding b
- 5. Critical exponents
- 6. The Future - Applying to my problem

0. Motivation

Numerical calculation: We can work only with finite size lattice

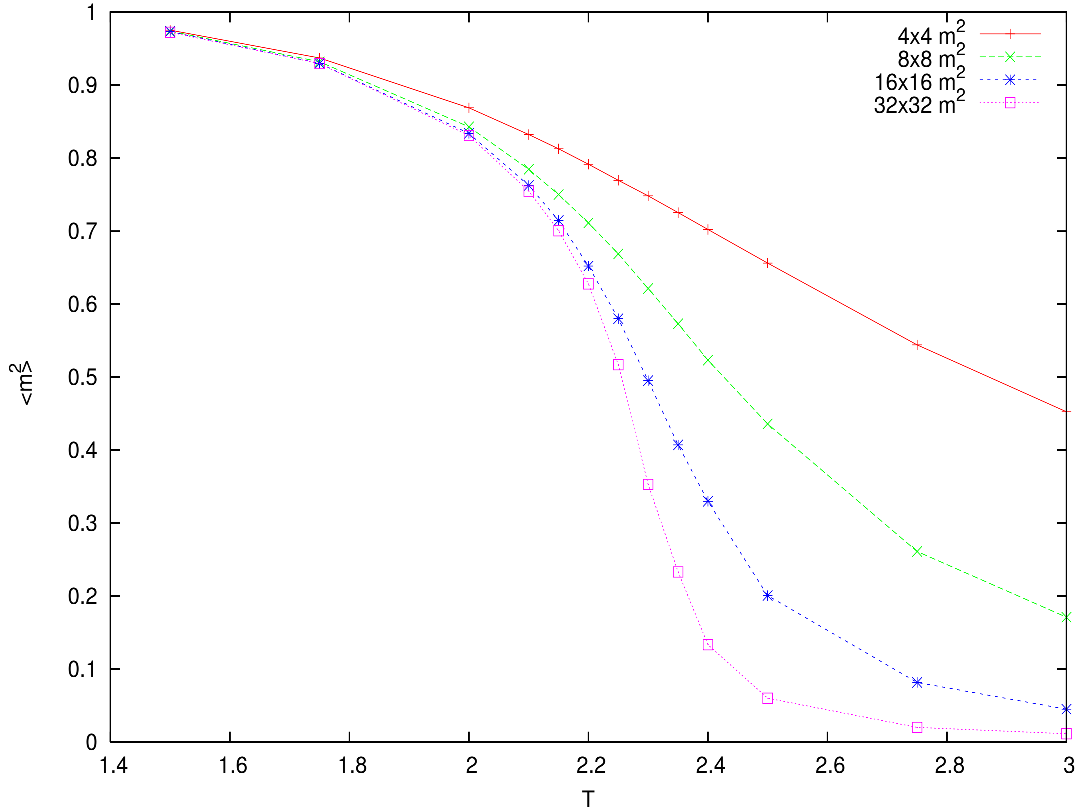


Figure 1: Plots of $\langle m^2 \rangle$ vs T for lattices of different sizes.

MC simulation of 2-D Ising model: Plots of (magnetization)^2 are *qualitatively* correct.

The problem: quantitative solution, for T_c and critical exponents.

1. Scaling hypothesis:

$$M^2(L,T) = L^a f[L^b(T-T_c)] \qquad (*)$$

M^2 - simulation data;
f - some (unknown) function

a, T_c, b - unknown

a, b - related to critical exponents ν, β
Show: $a = -\frac{2\beta}{\nu}$, $b = \frac{1}{\nu}$.

Why?
1) We believe L/ξ is the relevant variable (ξ - correlation length):
$\frac{L}{\xi} = L(T-T_c)^\nu = [L^{1/\nu}(T-T_c)]^\nu \qquad (**)$
Comparing (**) with (*) $\Rightarrow$ $b = \frac{1}{\nu}$.

2) As $L \to \infty$, $M^2 \sim (T-T_c)^{2\beta}$
$L^a f[L^b(T-T_c)] \sim (T-T_c)^{2\beta}$
$L^a[L^b(T-T_c)]^{2\beta} = L^{a+b\cdot 2\beta}(T-T_c)^{2\beta} \sim L^0(T-T_c)^{2\beta}$
$\Rightarrow a = -b\cdot 2\beta = -\frac{2\beta}{\nu}$

The procedure as follows.

0\. Get data for $M^2(L, T)$

Do MC simulations for lattices of different sizes.

1. Guess a value for a and

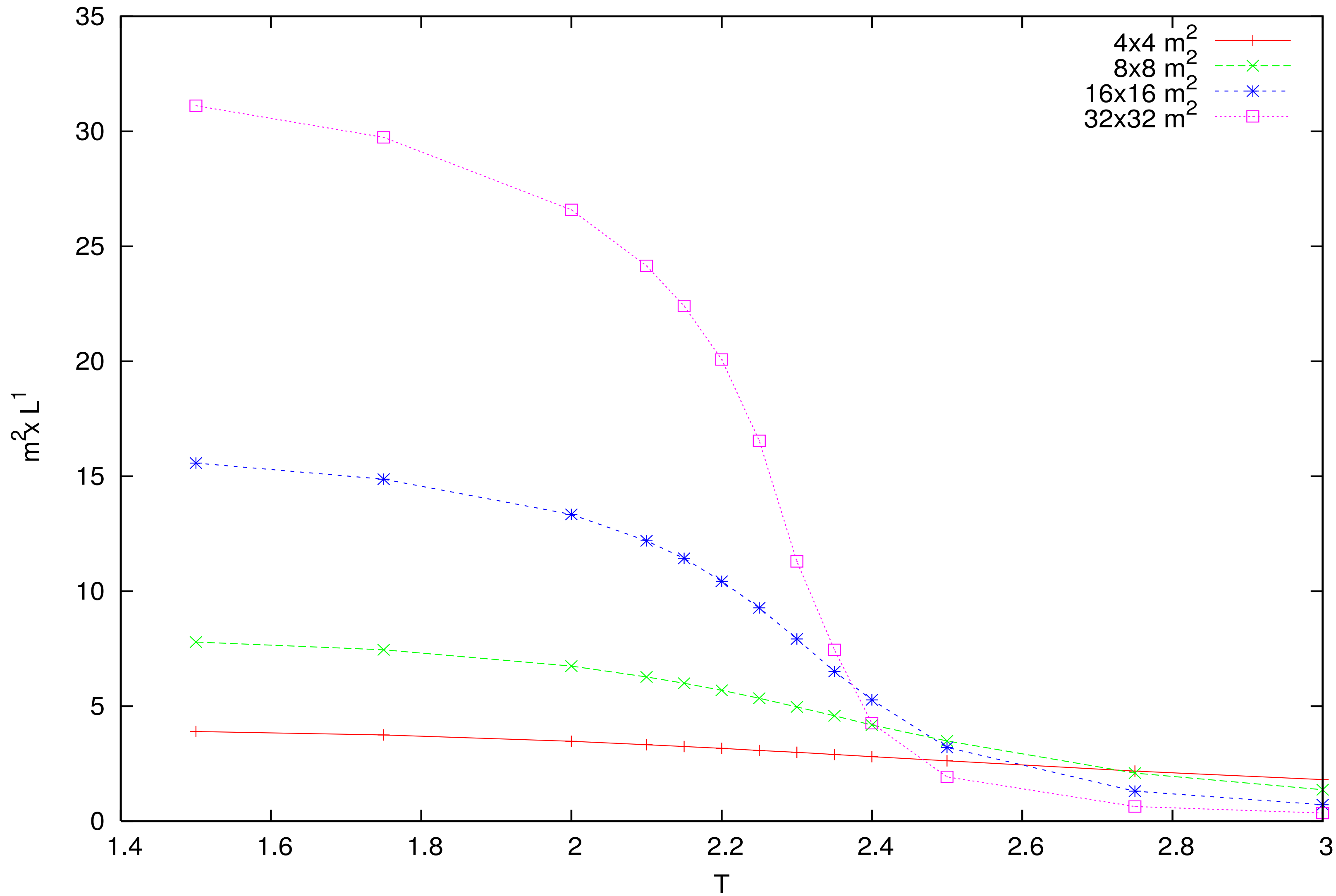
Plot $L^{-a} M^2(L,T)\ vs\ T$

All Curves must cross at $T = T_c$ - use this fact to get a.

Here's why:

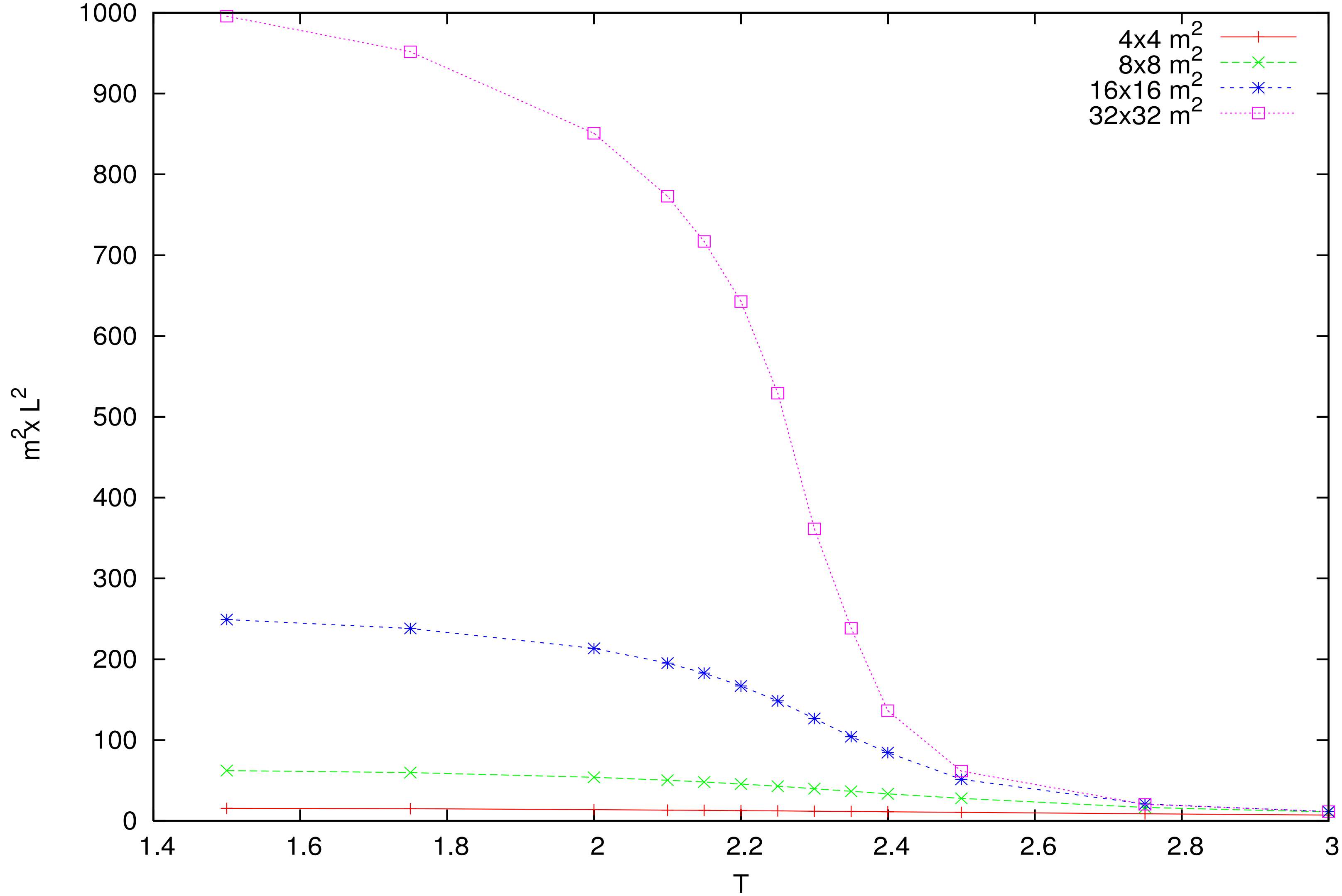
$$M^2(L,T) = L^a f[L^b(T - T_c)]$$
$$\Rightarrow\ L^{-a} M^2(L,T_c) = f[L^b(T_c - T_c)] = f(0)$$

Test: a=-1:



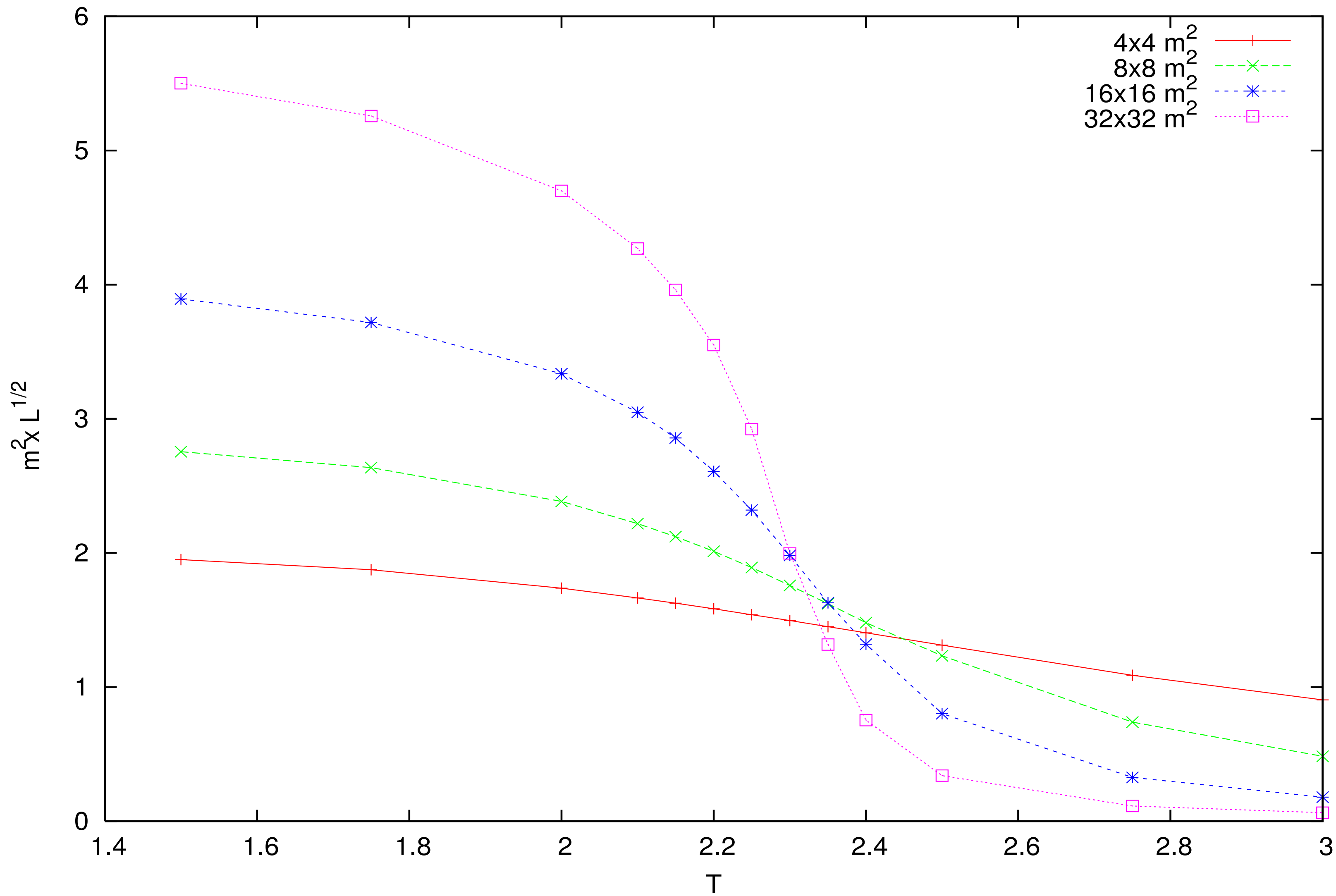
4x4 m2
8x8 m2
16x16 m2
32x32 m2
m2x L1
T
0
5
10
15
20
25
30
35
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

Test: a=-2:



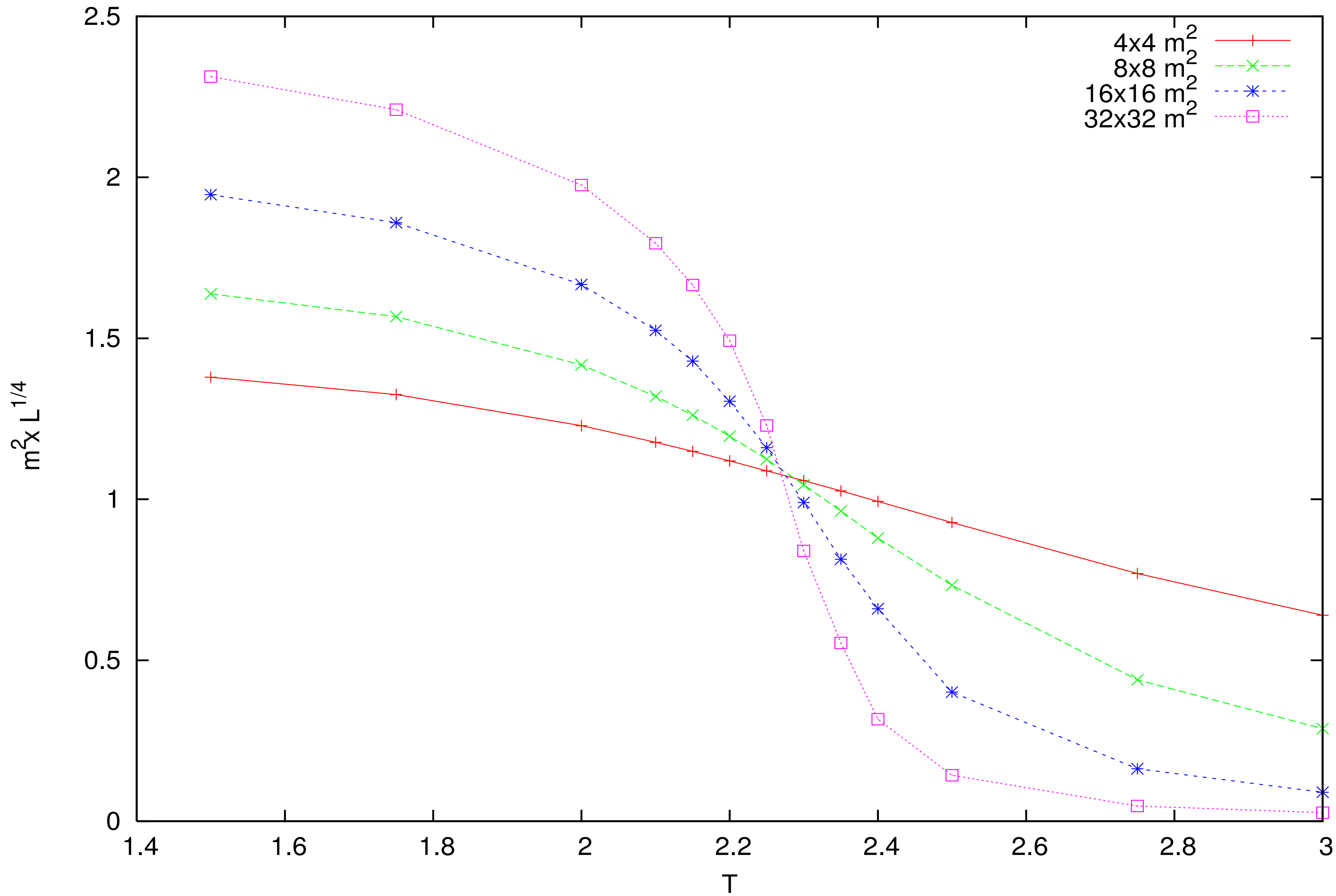
4x4 m^2
8x8 m^2
16x16 m^2
32x32 m^2
m^2 x L^2
T
1000
900
800
700
600
500
400
300
200
100
0
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

Test: a=-0.5:



4x4 m2
8x8 m2
16x16 m2
32x32 m2
m2x L1/2
T
0
1
2
3
4
5
6
1.4
1.6
1.8
2
2.2
2.4
2.6
2.8
3

Test: a=-0.25:



Match! (Correct $\beta = \frac{1}{8}$, $\nu = 1 \Rightarrow \; a = -\frac{2\beta}{\nu} = -\frac{1}{4}$)

$T_c = 2.265$ (Correct $T_c = 2.269$)

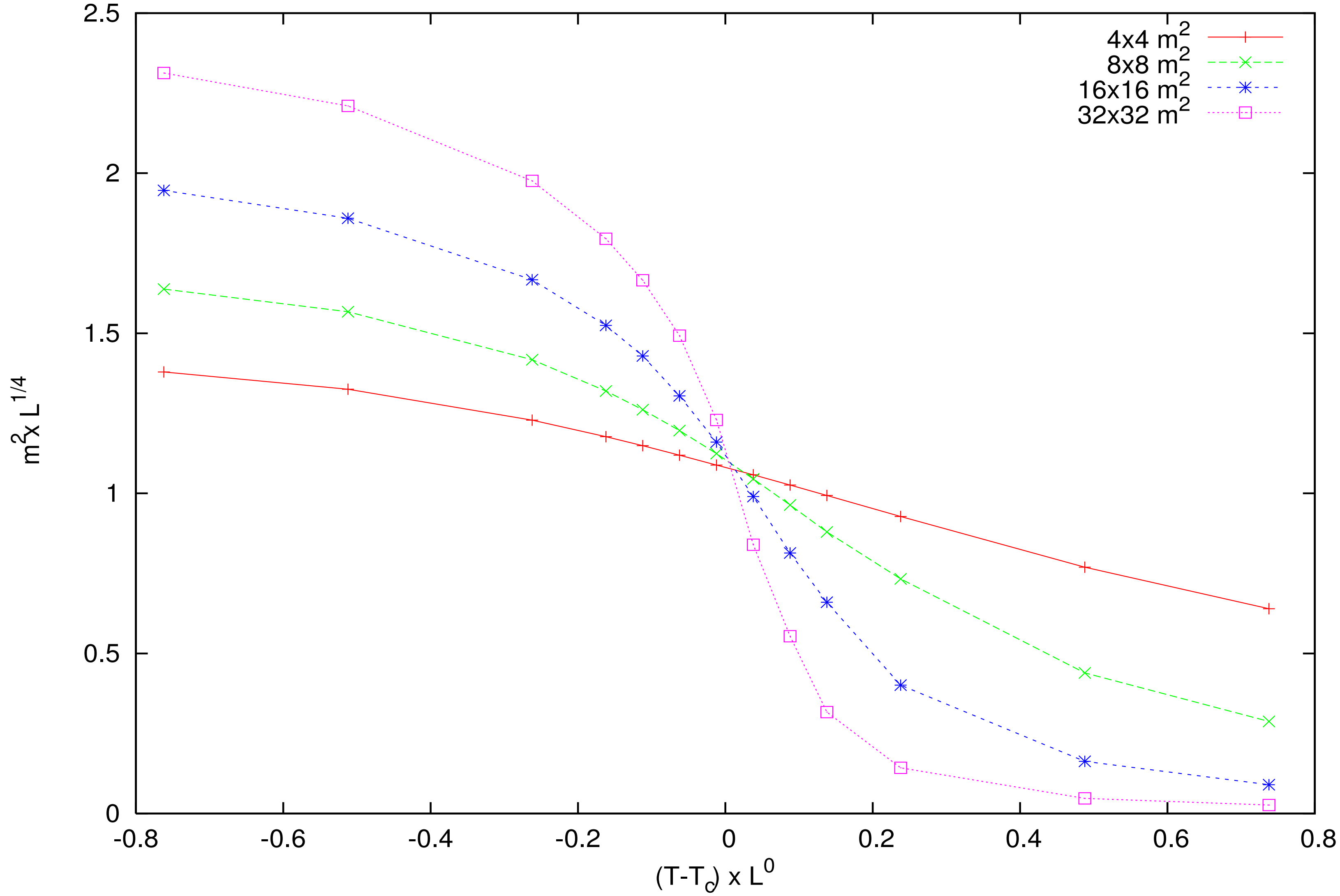
2. Now work on **b**

Guess b and

Plot $L^{-a} M^2(L,T) \; vs \; L^b(T - T_c)$

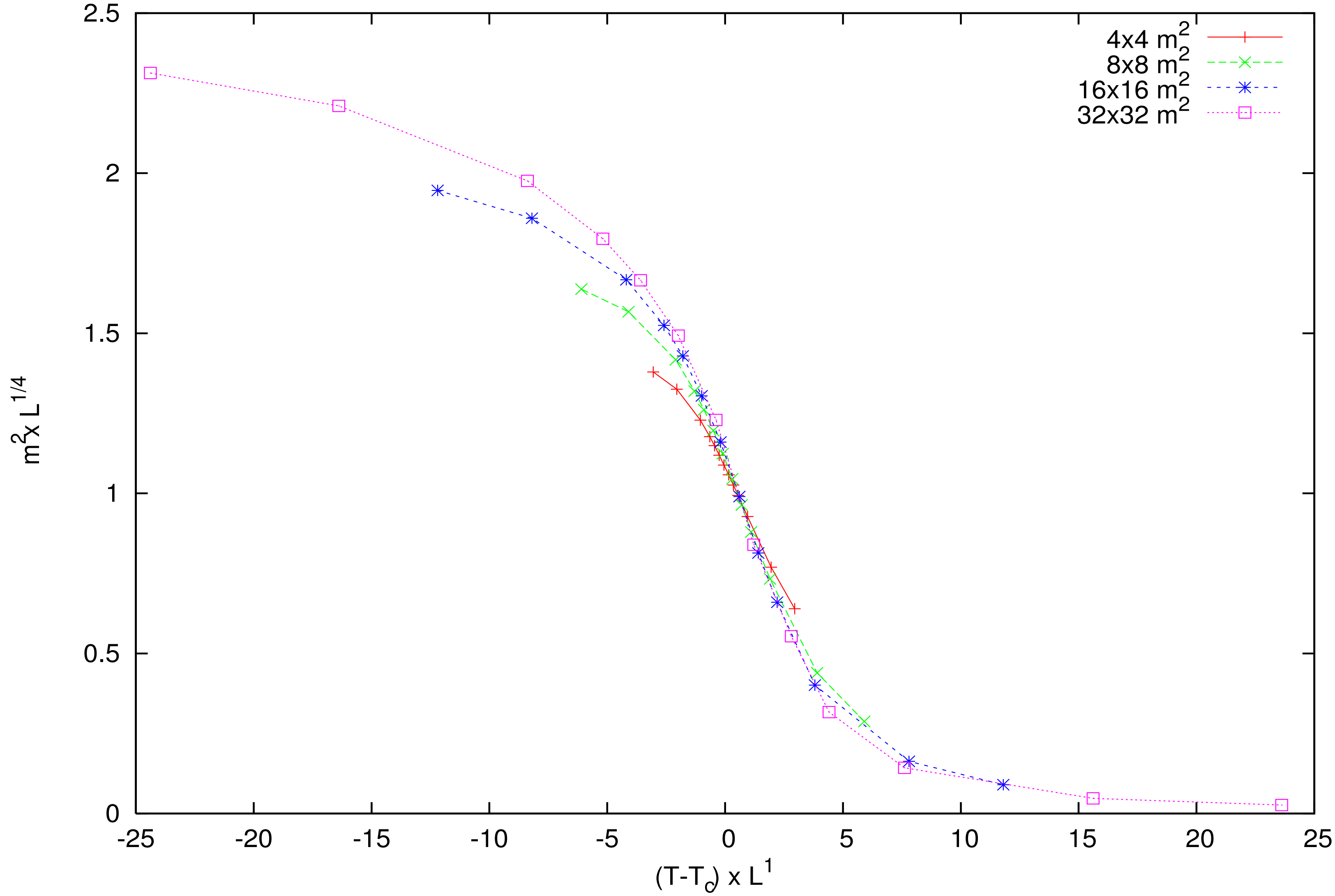
Curves must coincide around $T = T_c$

Test: b=0:



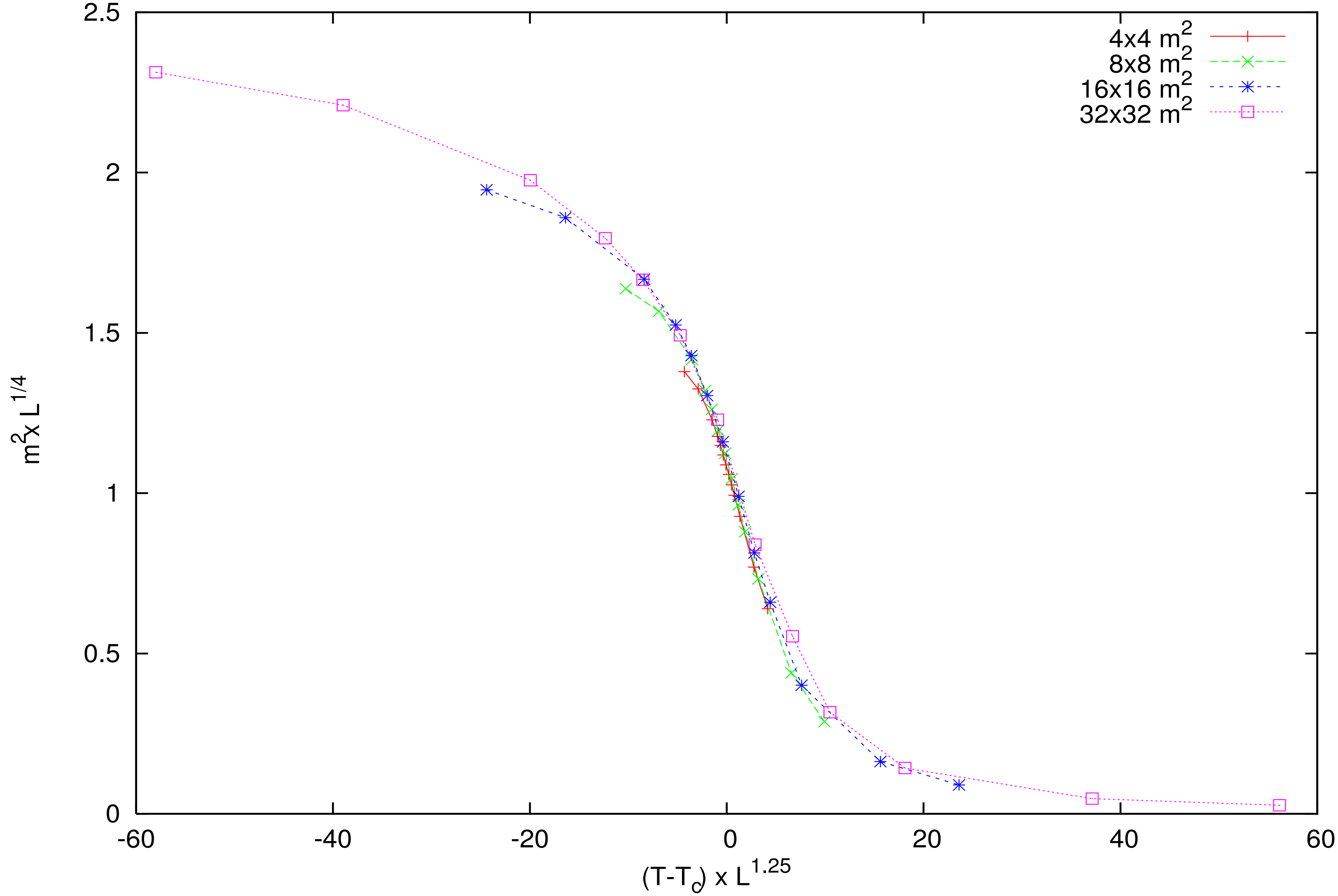
4x4 m^2
8x8 m^2
16x16 m^2
32x32 m^2
m^2 x L^1/4
(T-T_c) x L^0
2.5
2
1.5
1
0.5
0
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8

Test: b=1:



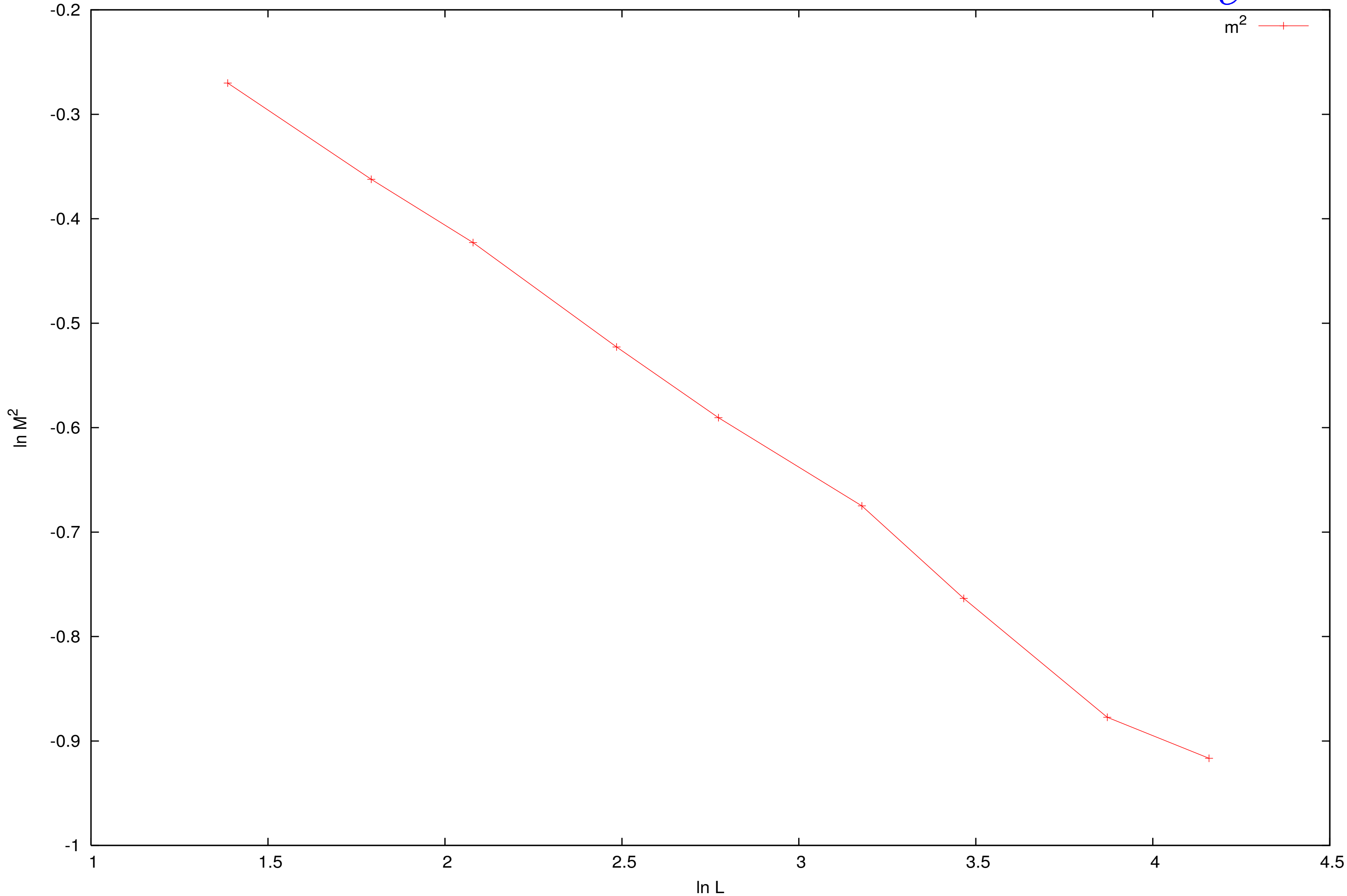
4x4 m^2
8x8 m^2
16x16 m^2
32x32 m^2
m^2 x L^1/4
(T-T_c) x L^1
2.5
2
1.5
1
0.5
0
-25
-20
-15
-10
-5
0
5
10
15
20
25

Test: b=1.25:



Not as good match as with a: The correct $\nu = 1 \Rightarrow b = \frac{1}{\nu} = 1$.

Plot of $\ln M^2$ vs $\ln L$ at $T = T_c$:



$M^2 = L^a f(0) \;\Rightarrow\; \ln M^2 = a \ln L + const$, the slope $= a = -\frac{2\beta}{\nu}$.

$a < 0$ is reasonable: $M^2 = 0$ at $T = T_c$ and $L = \infty$.

Critical exponents.

$$a = -\frac{2\beta}{\nu}, \quad b = \frac{1}{\nu}$$

We have

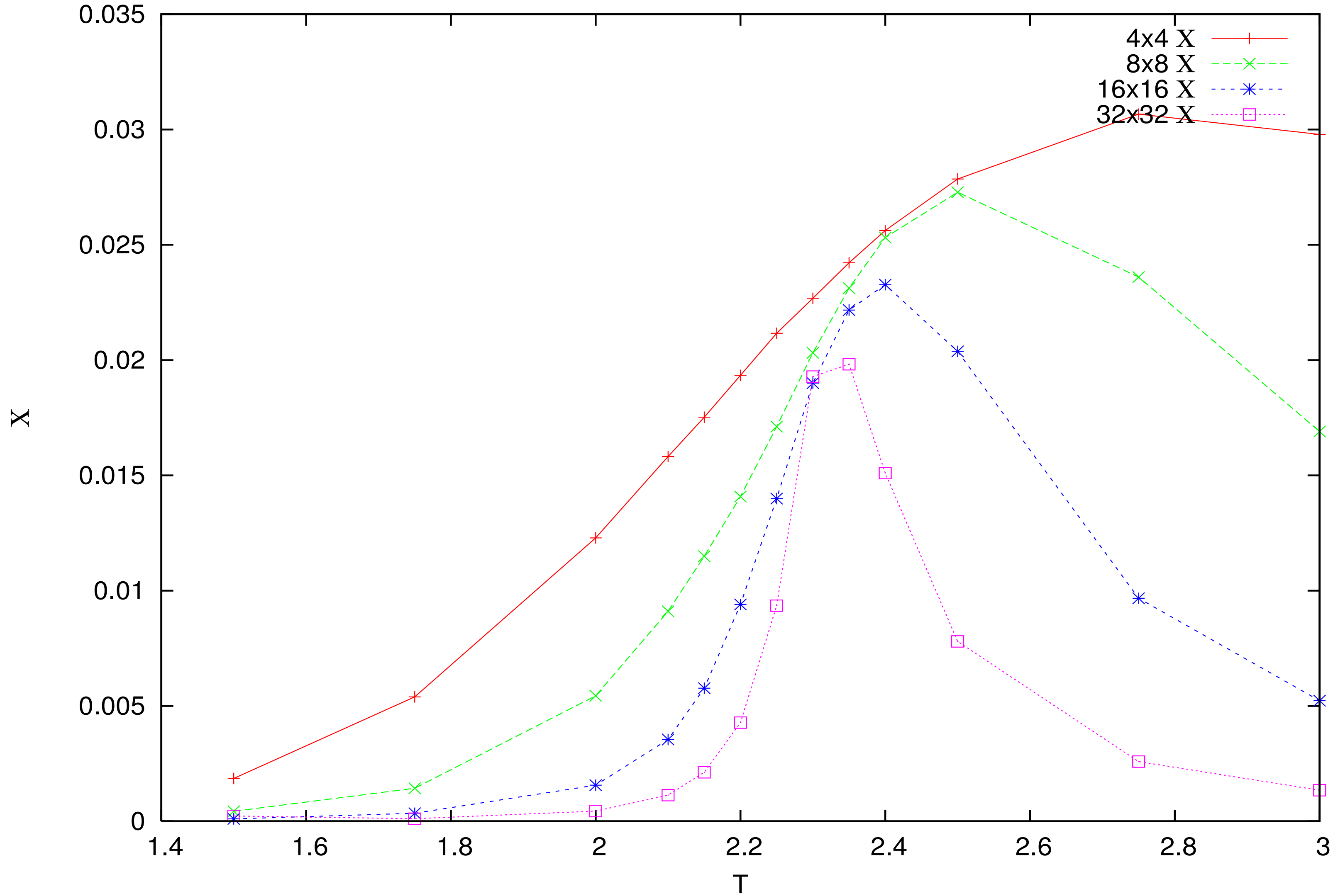
$$a = -\frac{1}{4}, \quad b = 1$$

$$\Rightarrow$$

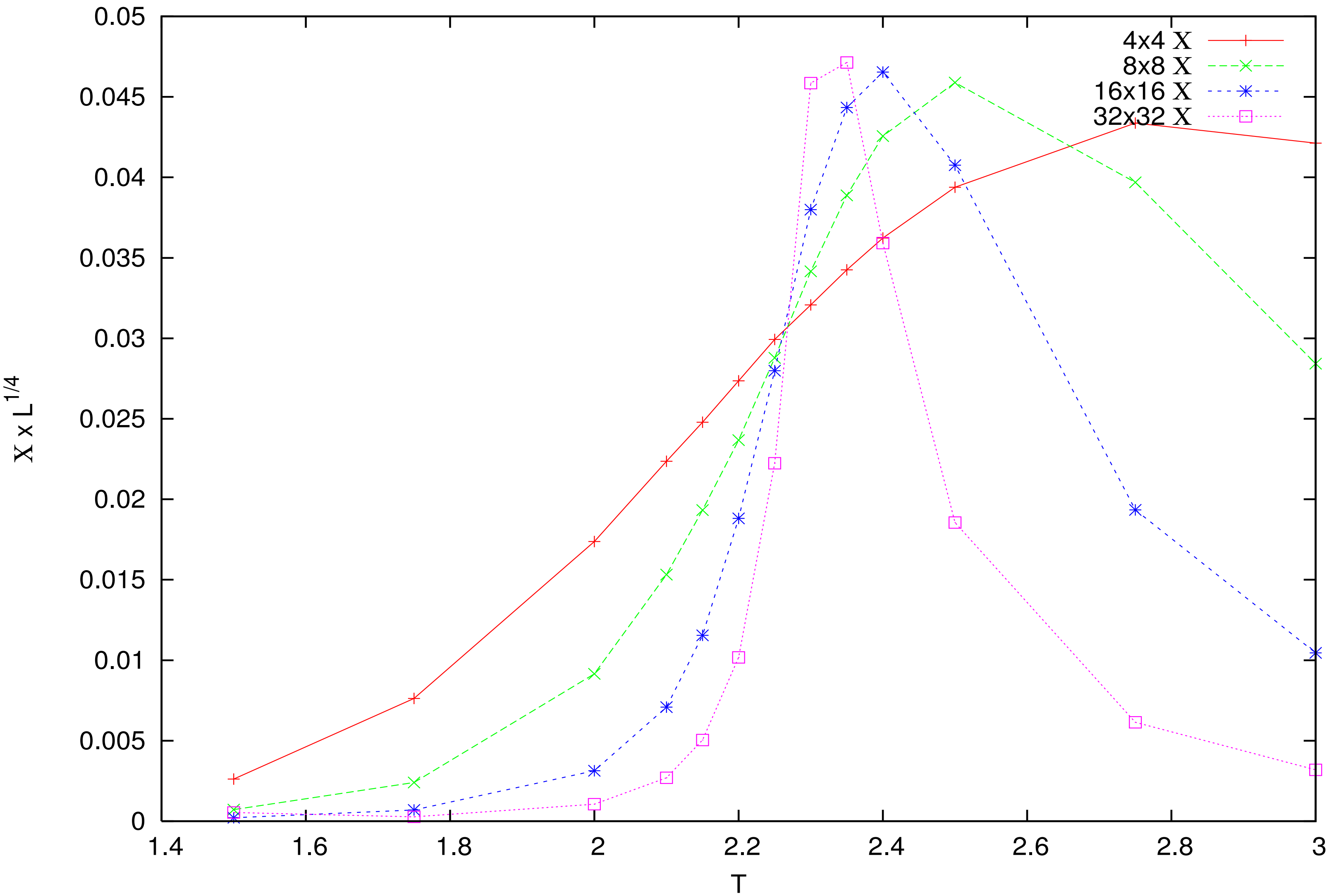
$$\nu = 1$$

$$\beta = \frac{1}{8}$$

Susceptibility: similarly

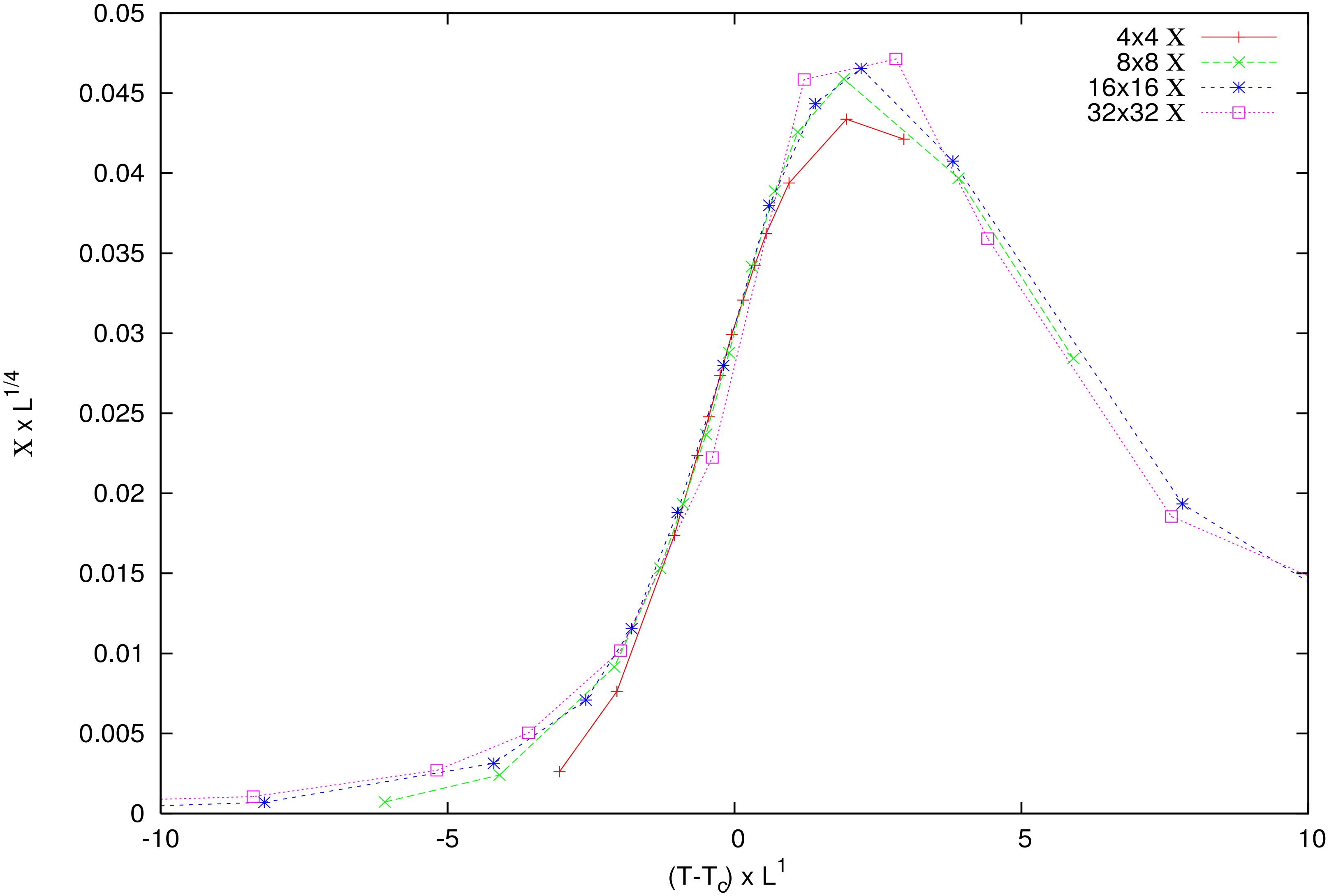


Guess a: Test: a=-0.25:



$\Rightarrow$ a=-0.25, $T_c = 2.265$

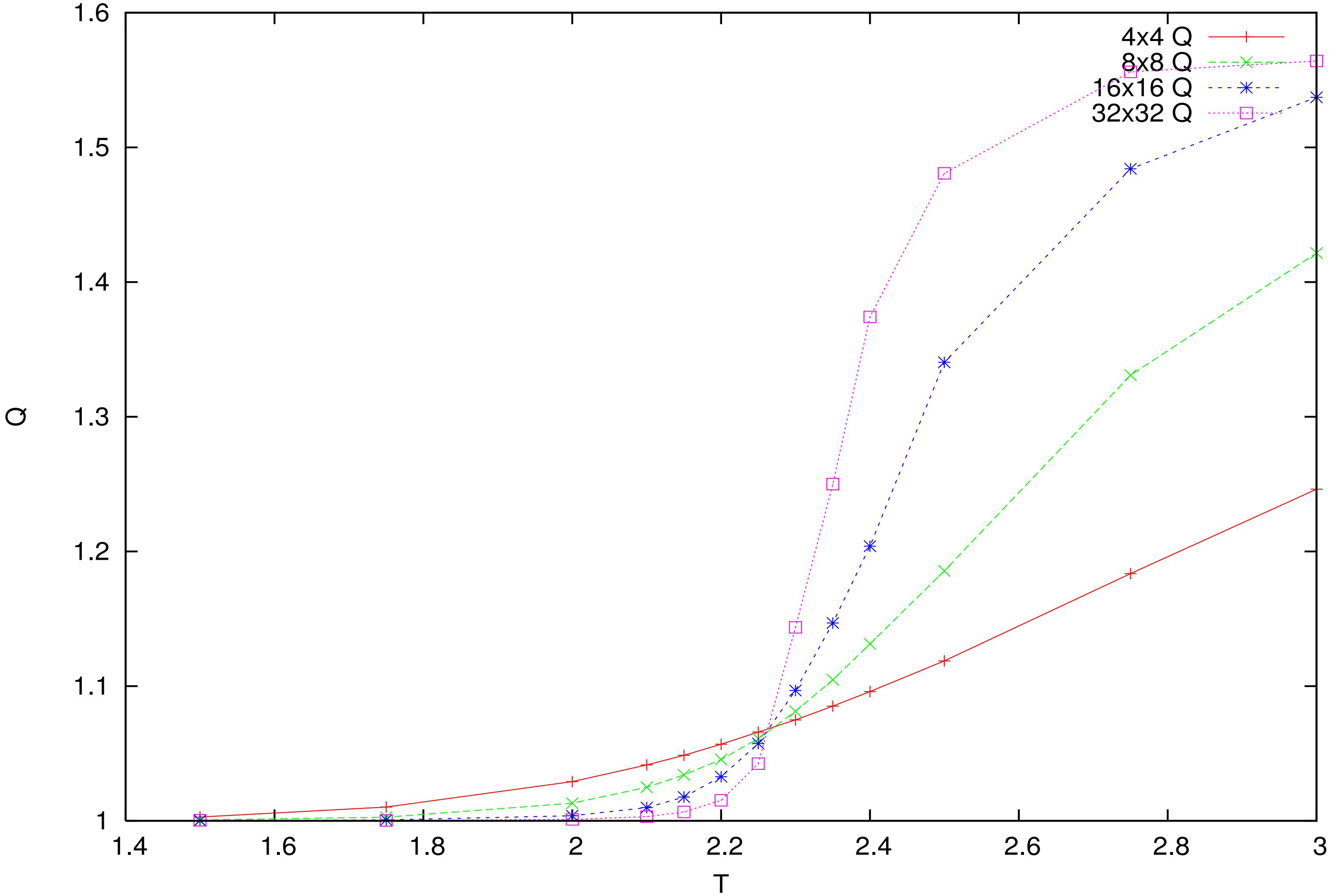
Guess b: Test: b=1:



$\Rightarrow$ a=-0.25, $T_c = 2.265$, b=1

The best way to find T_c: Binder ratio

$$Q = \frac{\langle M^2 \rangle}{\langle |M| \rangle^2}$$



Summary of the procedure.

0. Get data for $M^2(L,T)$

1. Find a: plots $L^{-a}M^2(L,T)$ *vs* T intersect.

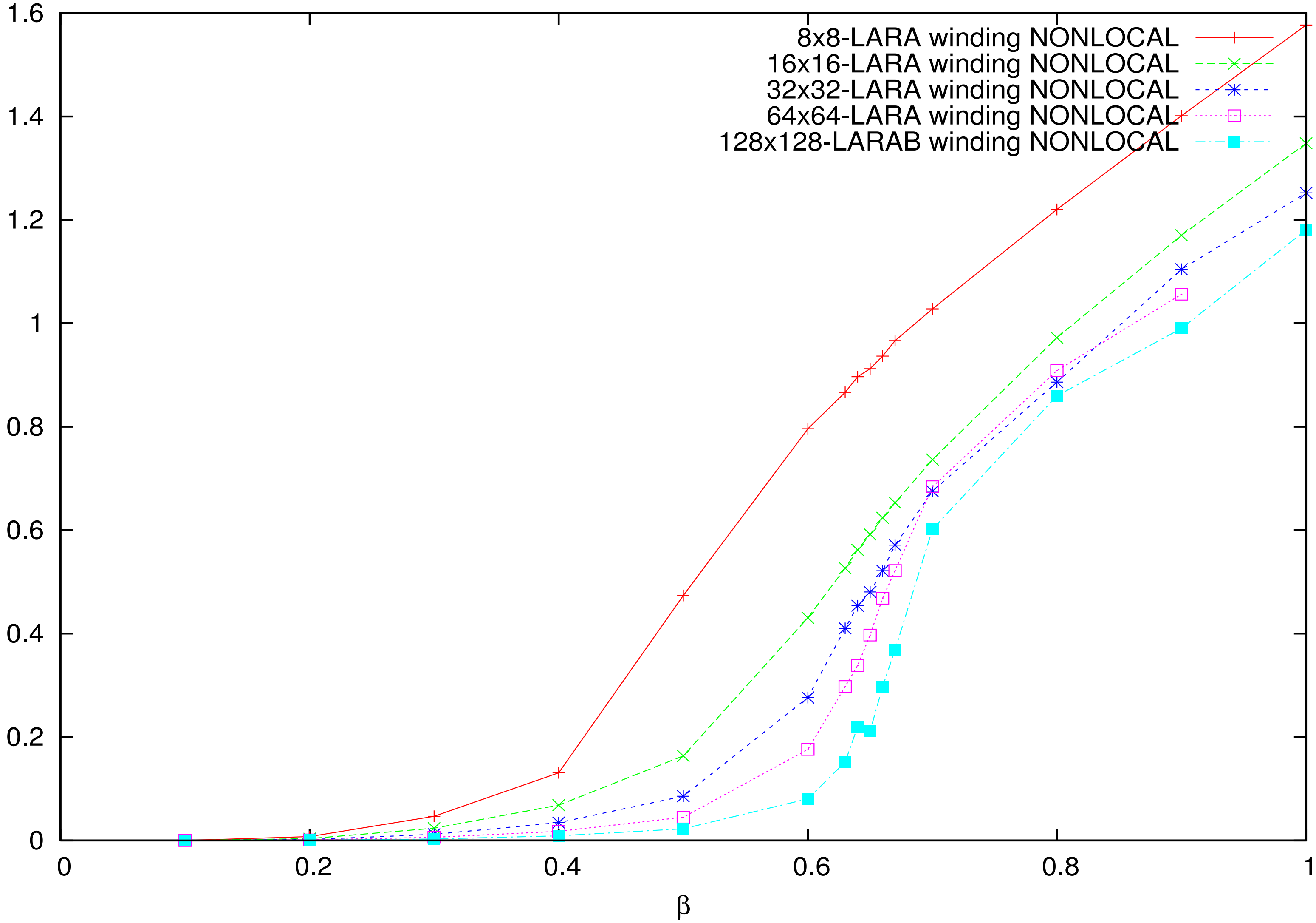
2. Find T_c: Curves cross at $T = T_c$.

3. Find b: plots $L^{-a}M^2(L,T)$ *vs* $L^b(T - T_c)$ coincide.

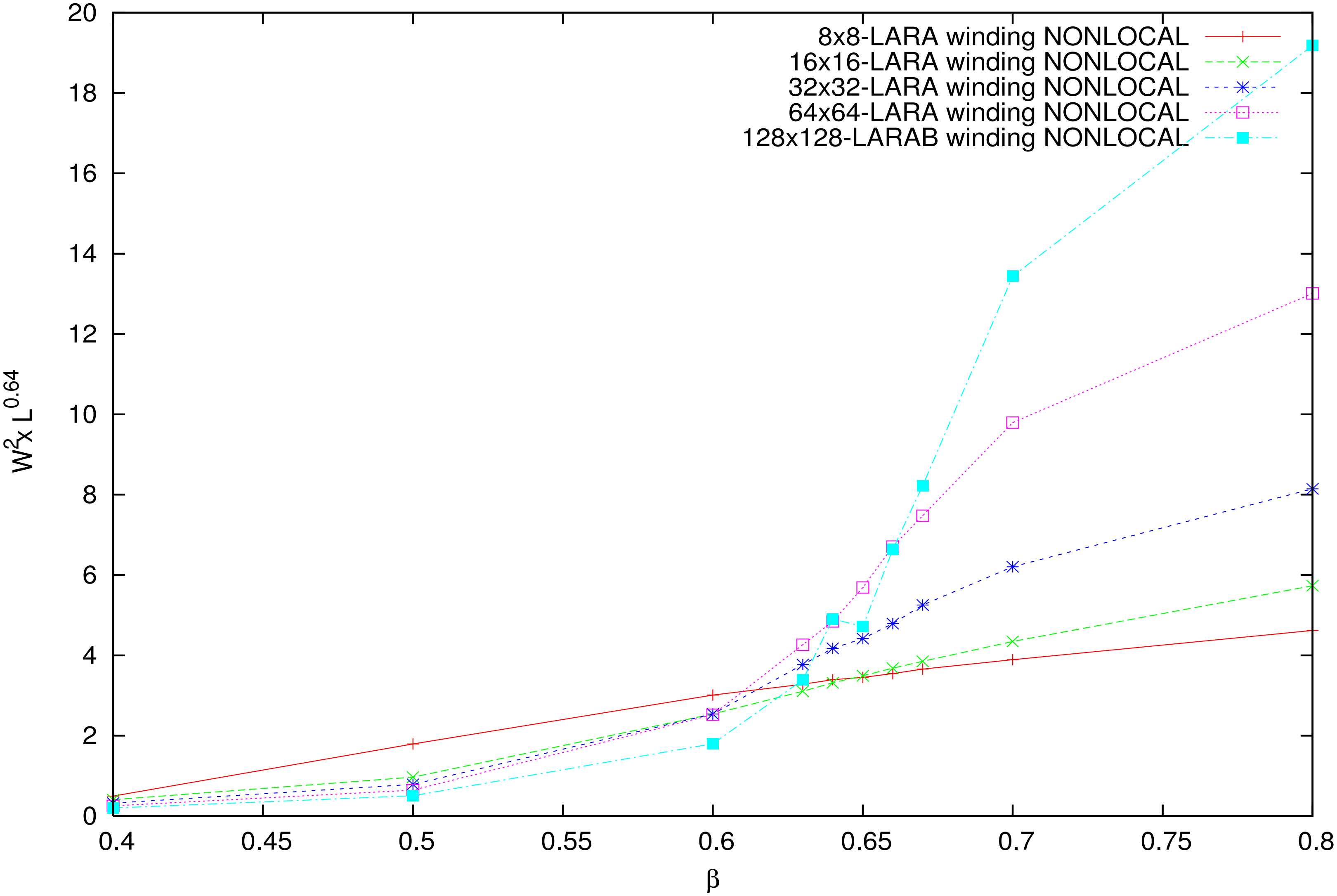
4. Find critical exponents: $\beta = -\frac{a}{2b}$, $\nu = \frac{1}{b}$.
Others from $d\nu = 2 - \alpha = 2\beta + \gamma = \beta(\delta + 1)$, $2 - \eta = \frac{\gamma}{\nu}$.

5. Have fun. Plot $M^2(L = \infty, T)$ *vs* T. Write a paper.

(Working) Apply to my problem: Feynman-Kikuchi model.



Test: a=-0.64:


8x8-LARA winding NONLOCAL
16x16-LARA winding NONLOCAL
32x32-LARA winding NONLOCAL
64x64-LARA winding NONLOCAL
128x128-LARAB winding NONLOCAL
$W^2 x L^{0.64}$
β
0
2
4
6
8
10
12
14
16
18
20
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8